# International Workshop of Knots and Links in a Spatial Graph Program and Abstracts

## July 20 (Tue) Arrival and registration (no workshop)

## July 21 (Wed)

10:00–10:40, 10:55–11:35 Blake Mellor (Loyola Marymount University)

**An overview of Robertson-Seymour-Thomas**

Abstract: I will give a broad overview of the proof of Sachs' linkless embedding conjecture by RST. My goal is simply to define the terminology, outline their main steps, and point out where they do or do not use topological arguments, to provide a starting point for the rest of the workshop.

11:50–12:30 Don Lawrence (Occidental College)

**Bohme's lemma**

Abstract: I will talk about sections 1 and 2 of RST's "Sachs' Linkless Embedding Conjecture" paper.

12:40–14:10 Lunch

14:10–14:50 Ramin Naimi (Occidental College)

**Panelled embeddings and fundamental groups, via the Scharlemann-Thompson theorem**

Abstract: I will talk about Section 3 of RST's 3rd paper, Sachs's Linkless Embedding Conjecture. Let's say an embedded graph satisfies the "$\pi_1$ Condition" if the complement of every subgraph of it in $S^3$ has free fundamental group. The main result of this section is: an embedded graph is panelled iff it satisfies the $\pi_1$ Condition. This relies on a similar theorem by Scharlemann and Thompson: an embedded planar graph is spherical iff it satisfies the $\pi_1$ Condition.

15:10–18:00 Working in groups

18:30– Reception party

## July 22 (Thu)

10:00–10:40 Timothy D. Comar (Benedictine University)

**Frame constructions, constructions for flat graphs, and the irreducibility of Sachs graphs**

Abstract: We will provide background about frame constructions and constructions for flat graphs and discuss how they are used in Roberston, Seymour, and Thomas' proof of Sachs' linkless embedding conjecture.

10:55–11:35 Makoto Ozawa (Komazawa University)

**Primitive spatial graphs and graph minors**

(joint work with Yukihiro Tsutsumi (Sophia University))

Abstract: We introduce a notion of primitive embeddings of a graph into the 3-sphere, and discuss about graph minors. This study is an approach to the knotless embedding problem.

11:50–12:30 Joel Foisy (SUNY Potsdam)

**A survey of recent result on intrinsically knotted and linked graphs**

Abstract:

12:40–14:10 Lunch

14:10–18:00 Working in groups

## July 23 (Fri)

10:00–10:40 Erica Leigh Flapan (Pomona College)

**Intrinsic knotting and linking of complete graphs**

Abstract: We show that for every $m \in \mathbf{N}$, there exists an $n \in \mathbf{N}$ such that every embedding of the complete graph $K_n$ in $\mathbf{R}^3$ contains a link of two components whose linking number is at least $m$. Furthermore, there exists an $r \in \mathbf{N}$ such that every embedding of $K_r$ in $\mathbf{R}^3$ contains a knot $Q$ with $|a_2(Q)| \geq m$, where $a_2(Q)$ denotes the second coefficient of the Conway polynomial of $Q$.

10:55–11:35 Hugh Nelson Howards (Wake Forest University)

**Knotted graphs in balls**

Abstract: We look at when graphs contained in balls are knotted.

11:50–12:30 Kouki Taniyama (Waseda University)

**Higher dimensional links in a simplicial complex embedded in a Euclidean space**

Abstract: The Conway-Gordon theorem for the complete graph on six vertices has natural higher dimensional generalization. This talk is related to this topics.

12:40–14:10 Lunch

14:10–18:00 Working in groups

## July 24 (Sat) Excursion

## July 25 (Sun)

10:00–10:40 Hiromasa Moriuchi (Osaka City University)

**An enumeration of theta-curves and handcuff graphs**

Abstract: We enumerate all the prime $\theta$-curves with up to seven crossings, and all the handcuff graphs with up to six crossings. We can enumerate all the prime $\theta$-curves (or handcuff graphs) in order of crossing numbers by using a prime basic $\theta$-polyhedron. A $\theta$-polyhedron is a connected planar graph embedded in 2-sphere, whose two vertices are 3-valent, and the others are 4-valent. There exist twenty-four prime basic $\theta$-polyhedra with up to seven 4-valent vertices. We can obtain a $\theta$-curve diagram (or handcuff graph diagram) from a prime basic $\theta$-polyhedron by substituting algebraic tangles for their 4-valent vertices.

10:55–11:35 Yukihiro Tsutsumi (Sophia University)

**Strong $n$-triviality of spatial graphs**

Abstract: Let $L$ be a link in $S^3$. We say that $L$ is *strongly $n$-trivial* if it admits a diagram such that for some set $V$ of $n+1$ crossing points, changing the under/over information on every non-empty subset of $V$ yields a trivial link. A link is called a *boundary link* if the components bound mutually disjoint Seifert surfaces. It is known that strongly $n$-trivial links are boundary links. However this statement does not hold for spatial graphs. In this talk, we discuss surfaces bounded by cycles of spatial graphs and the notion of strong $n$-triviality of spatial graphs.

11:50–12:30, 14:10–14:50 Ryo Nikkuni (Waseda University)

**A survey on the regular projection of spatial graphs**

Abstract: Study of the regular projection of spatial graphs is more rich in content as compared with the regular projection of knots and links. Actually some interesting phenomena in the regular projection of spatial graphs which are not appeared in the one of knots and links have been discovered Since 1990's. In this talk we make a survey on the research in the regular projection of spatial graphs and present the unsolved problems.

12:40–14:10 Lunch

15:10–18:00 Working in groups

## July 26 (Mon)

10:00–10:40 Thomas R. Fleming (U C San Diego)

**Intrinsically linked graphs with Novel properties**

Abstract: We construct a graph $G$ such that any embedding of $G$ into the three sphere contains a nonsplit two component link where at least one of the components is a non-trivial knot. We will also prove that every embedding of $K_{10}$ contains a nonsplit two

component link with even linking number.

10:55–11:35 Chbili Nafaa (Tokyo Institute of Technology)

**The Yamada skein modules and symmetry of spatial graphs**

Abstract: Let $\mathcal{R} = \mathbf{Z}[A^{\pm 1}, \delta^{-1}]$, where $\delta = -A^2 - A^{-2}$. Let $M$ be a three-manifold and let $\mathcal{G}$ be the set of all isotopy classes of graphs embedded in $M$. We define the Yamada skein module of $M$ as the quotient of the free module $\mathcal{R}[\mathcal{G}]$ by the skein relations introduced by S. Yamada to define the topological invariant of spatial graphs known as the Yamada polynomial. In this talk, We compute the Yamada skein module for handelbodies and explore its relationship with the Kauffman bracket skein module (computed by Przytycki and Bullock). In particular, we prove that the Yamada skein module and the Kauffman bracket skein module of the solid torus are isomorphic. We use the skein module of the solid torus to study the $\mathbf{Z}/p\mathbf{Z}$-symmetry of spatial graphs. Namely, we introduce necessary conditions for a spatial graph to be periodic.

11:50–12:30 Miyuki Okamoto (Nippon Institute of Technology)

**On the complete list of $T$-minimal graphs up to 9 vertices**

(joint work with Kazuaki Kobayashi (Tokyo Woman's Christian University))

Abstract: An intrinsically chiral graph $G$ is $T$-minimal if all (proper) minor graphs of $G$ are achirally embeddable. In this talk, we give the complete list of $T$-minimal graphs up to 9 vertices.

12:40–14:10 Lunch

14:10–18:00 Working in groups

## July 27 (Tue)

10:00–10:40 Reiko Shinjo (Waseda University)

**Bounding disks to a spatial graph**

Abstract: We consider Seifert surfaces of knots in a spatial graph whose interiors are mutually disjoint. We give the upper bound of the number of such surfaces. Then we show that for a given abstract graph there is a spatial embedding of the graph which realizes the upper bound.

10:55–11:35, 11:50–12:30 Ryo Nikkuni (Waseda University)

**Invariants and local moves on spatial graphs**

Abstract: We explain recent results about the classification of spatial embeddings of a graph up to several equivalence relations generated by specific local moves and ambient isotopies.

12:40–14:10 Lunch

14:10–18:00 Working in groups

18:30– Banquet

**July 28 (Wed)** Departure (no workshop)