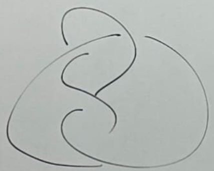
[TTU] Tange Tran U. $\cdots$ §1

2021 IMRN

[BTTU] Benard + T $\cdots$ §2 $\sim$ 4

§1 1.1 Char. Var

$k = 4_1$,



$a, b$ : meri's

$w := [a, b^{-1}]$

$\pi_K = \langle a, b \mid aw = wb \rangle$

$g \in \pi_K$

$$\mathrm{tr}_g : \mathrm{Hom}(\pi_K, SL_2(\mathbb{C})) \to \mathbb{C}$$

$$\rho \longmapsto \mathrm{tr}\,\rho(g)$$

$x := \mathrm{tr}\,a$, $y := \mathrm{tr}\,ab$

$$\mathrm{Hom}(\pi_K, SL_2(\mathbb{C})) /\!/ \mathrm{Conj} = \{\mathrm{tr}\,\rho \mid \rho \in \mathrm{Hom}\}$$

$$\{\text{irr. char's}\} \xleftrightarrow[1:1]{} \{f = 0\}$$

$f = y^2 - (x^2+1)y + 2x^2 - 1 \in \mathbb{Z}[x,y]$

## 1.2 torsion function

$\rho^R$ : Riley's univ. $SL_2$-rep

$\tau_{\rho^R} = \prod_i (\mathrm{ord}\, H_i(\pi, \rho^R))^{(-1)^i}$

$\tau(x,y) \in \mathbb{Z}[x,y]$

$\rho : SL_2\mathbb{C}$ rep, irr

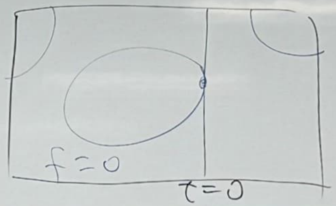
$\tau_{[\rho]} = 0 \overset{}{\underset{\text{iff}}{\Longleftrightarrow}} \rho$ : non-acyclic (i.e. $H_i(\rho) \neq 0, \forall i$)

Rem $\rho_1 : GL_1$ univ rep $\Rightarrow \tau_{[\rho_1]} = \dfrac{\Delta_K(t)}{t-1}$

(B. Mazur: the multiplicities of zeros of $\tau$ : subtle & interesting!)

$k = 4, \quad \tau = -2(x-1)$

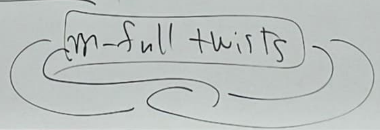
☆ $\begin{cases} f = 0 \\ \tau = 0 \end{cases}$ は共有点で接する




## 1.3 Zeros

$\tau$ on $\{f=0\}$ の zeros は mult = 2

Thm $k = J(2,2m)$, $\forall m$ でも成立.



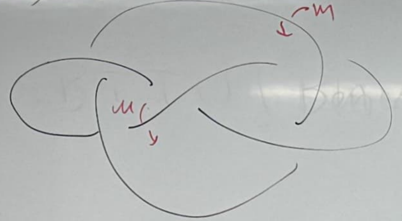

(Rem 他の prime knots でも check できている)

## 1.4 Geom interpret?

Thm $J(2,2m)$ の irr rep は non-acyclic

$\Longleftrightarrow$ $-3$-filling を経由 かつ ある元が ord = 3.

## §2 $W$




$\pi = \langle m, \mu \mid m\omega = \omega\mu \rangle$

$\omega = \omega(m, \mu)$

$x = \mathrm{tr}\, m, \quad y = \mathrm{tr}\, \mu$

$z = \mathrm{tr}\, m\mu$

$$\begin{cases} f = xyz^2 - z(x^2+y^2+z^2) + xy + 2z \\ \tau = -2(x+y-z-2) \end{cases}$$

**Thm**

① $\begin{cases} f = 0 \\ \tau = 0 \end{cases} \iff \begin{cases} x+y-1 = 0 \\ z = -1. \end{cases}$

$\Longleftarrow$ $\rho$は$(-3,-3)$-fillingを経由

Rem the result $W(-3,-3) = L(3,1) \# L(3,1)$

② $\left(\mathbb{C}[x,y,z]/(f,\tau)\right)_{(P)}$    $P$: a generic pt on $f=\tau=0$

$\cong \left(\mathbb{C}[x,y,z]/((x+y-1)^2, z+1)\right)_{P}$.

"mult = 2" (= the length of a local ring)

いいかえ $f=0$ の $(a,b,c)$ まわりでの
陰関数 $z = z_f(x,y)$

$L(x,y) := \tau(x,y,z_f(x,y))$

$(a,b)$ での Taylor 展開は
$\in \mathfrak{m}^2, \notin \mathfrak{m}^3$

$\mathfrak{m} = (x-a,\ y-b) \subset \mathbb{C}[[x-a,\ y-b]]$.

## §3 Geom.

**Thm** Assume:
• $M$ hyp 3-mfd / cusp
$\downarrow$ filling
• $N$ Seif. fib mfd

$\chi$ (base orbifold) $\neq 0$

• 
$$\begin{tikzcd} \rho: \pi_1 M \arrow[r] \arrow[d] & SL_2\mathbb{C} \\ \pi_1 N \arrow[ur] \arrow[u, phantom, "\circlearrowleft"] & \end{tikzcd}$$

(i.e. exceptional surj を経由)

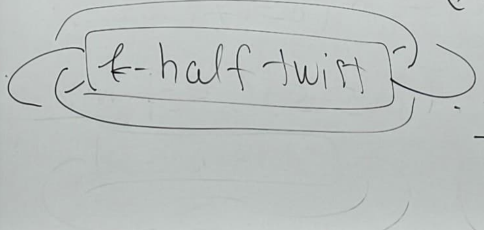
• $\rho$: non-triv on $\partial$
• generic fiber $\mapsto$ id

Then, $\rho$: non-acyclic

(pf a ver of [Kitano 1994] for $\partial M \neq \emptyset$ cases.)

## §4 $W_k$

(If $k$: odd) Borromean ring
$\swarrow$ filling




$\rightsquigarrow J(k, 2\ell)$
$-1/\ell$-filling

**Thm** "mult $= 2$"

$\because$ $v = x^2+y^2+z^2-xyz-2$
$f, \tau$

$$\left(\mathbb{C}[x,y,z,v]/(f, \tau, v-(\dots))\right)_{(p)} \cong \mathbb{C}[x,y]\Big/\left(z-(\dots),\ xy-(\dots),\ \left(x+y-\frac{v+2}{T_n(v)-2}\right)^2\right)$$