結び目のシャドーコサイクル不変量のサテライト化公式

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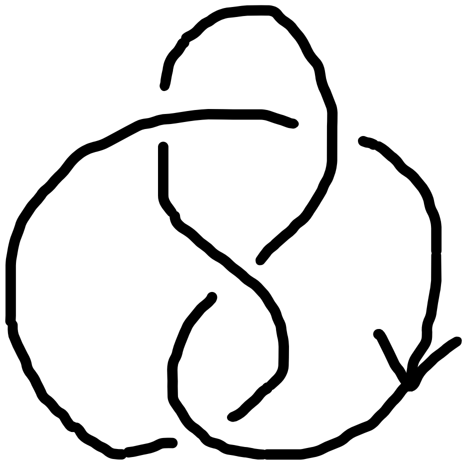
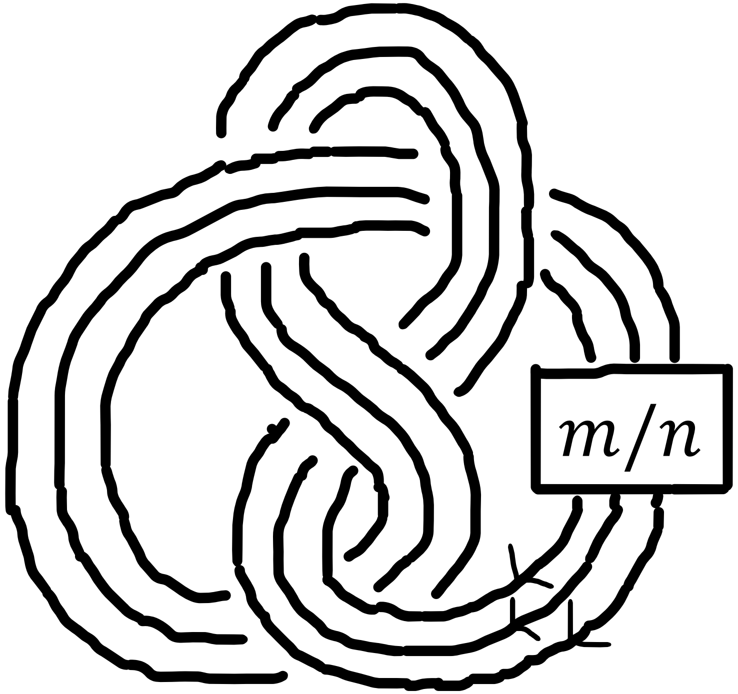
§0. Intro





結び目の
(n,m)-ケーブル化

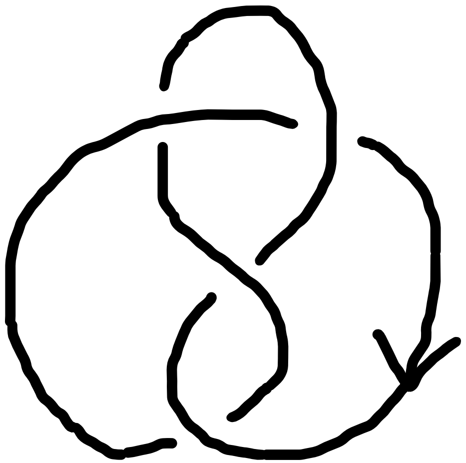
$\frac{m}{n}$-twist

φ, θ:カンドルコサイクル

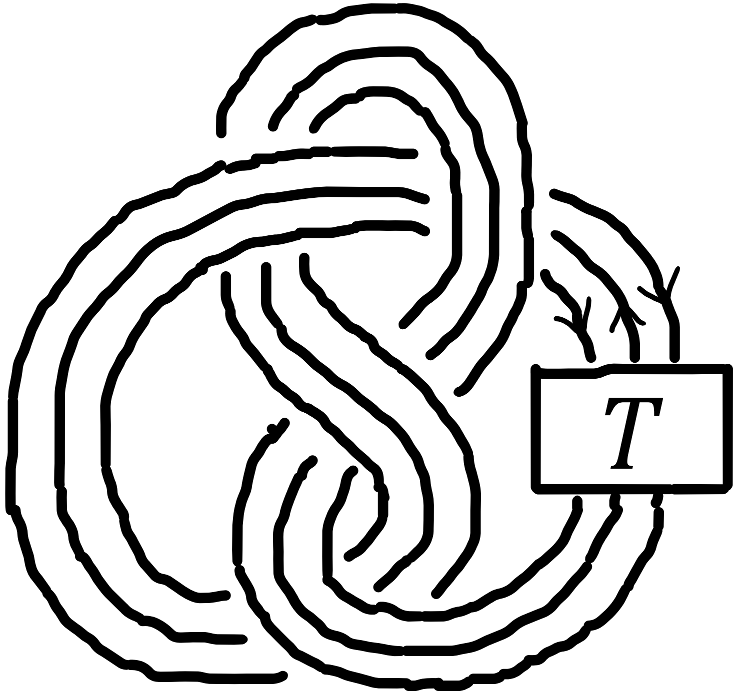
カンドルコサイクル不変量 $\Phi_\varphi\left(K^{(n,m)}\right) = \sum c_* \Phi_*(K)$ [Naruse'15] [Ishikawa'16]

シャドーコサイクル不変量 $\Psi_\theta\left(K^{(n,nk)}, r\right) = \sum c_* \Psi_*(K)$ [Naruse'15]

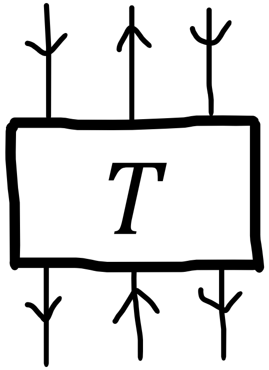
§0.



K

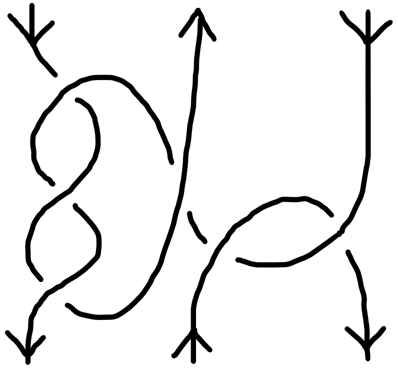


サテライト結び目$K^{(3)}(T)$



$(+,-,+)$-タングル

例



目標

具体的な(X,θ)に対するサテライト化公式の導出

$$\Psi_\theta\left(K^{(3)}(T)\right) = (T\text{の不変量と}K\text{の不変量で表示})$$

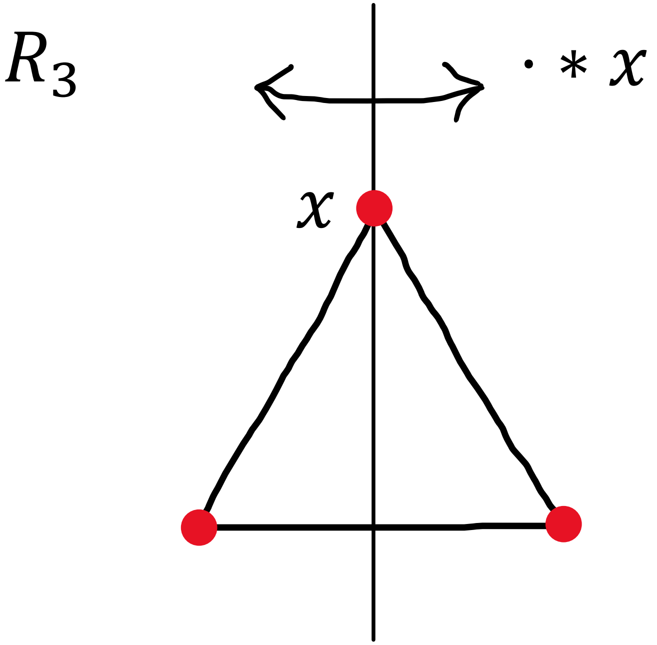
(2面体カンドルR_p, 8面体カンドルQ_6)

目次

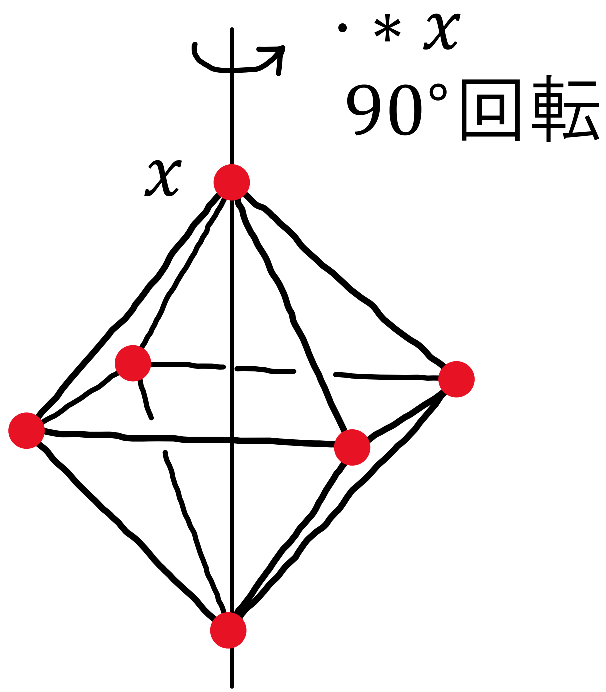
§1. 準備

$(X,*)$がカンドル $\Leftrightarrow \begin{cases} x * x = x \\ \exists!\, a * y = z \\ (x * y) * z = (x * z) * (y * z) \end{cases}$

例　2面体カンドル$R_n = (\mathbb{Z}_n,*)$



例　8面体カンドルQ_6



§1.

X: カンドル　A: アーベル群

$\theta: X^3 \to A$がカンドル3-コサイクル

$$\Leftrightarrow \begin{cases} \theta(w,y,z) - \theta(w,x,z) + \theta(w,x,y) \\ = \theta(w*x,y,z) - \theta(w*y,x*y,z) + \theta(w*z,x*z,y*z), \\ \theta(x,y,y) = 0, \\ \theta(x,x,y) = 0, \end{cases}$$

例　R_p(p: 奇素数)のMochizuki 3-コサイクル $\phi: (R_p)^3 \to \mathbb{Z}_p$

$$\phi(w,x,y) = (w-x)\frac{x^p + (2y-x)^p - 2y^p}{p} \in \mathbb{Z}_p$$

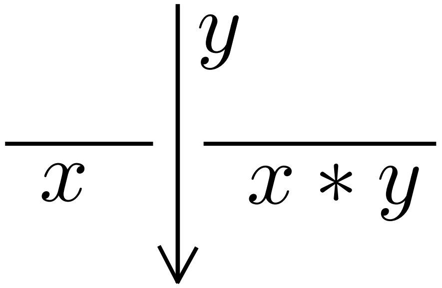
例　$\phi_6: (Q_6)^3 \xrightarrow{f\times f\times f} (R_3)^3 \xrightarrow{\phi} \mathbb{Z}_3$

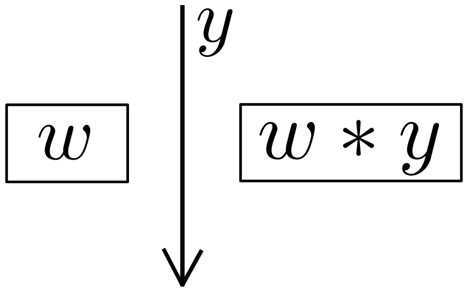
$f: Q_6 \twoheadrightarrow R_3$

$H^3(R_p;\mathbb{Z}_p) \cong \mathbb{Z}_p$

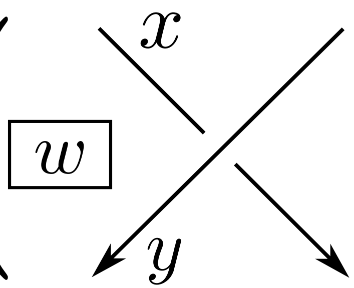
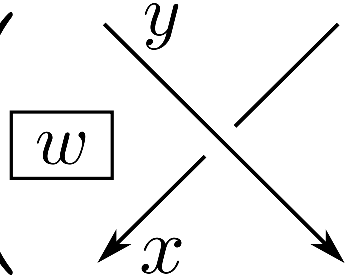
$H^3(Q_6;\mathbb{Z}_3) \cong \mathbb{Z}_3$

§1. X coloring $\mathcal{C}$: $\{D$の$arc\} \to X$　　　shadow X coloring: $\{D$の領域$\} \to X$





3-コサイクル
$\theta: X^3 \to A$

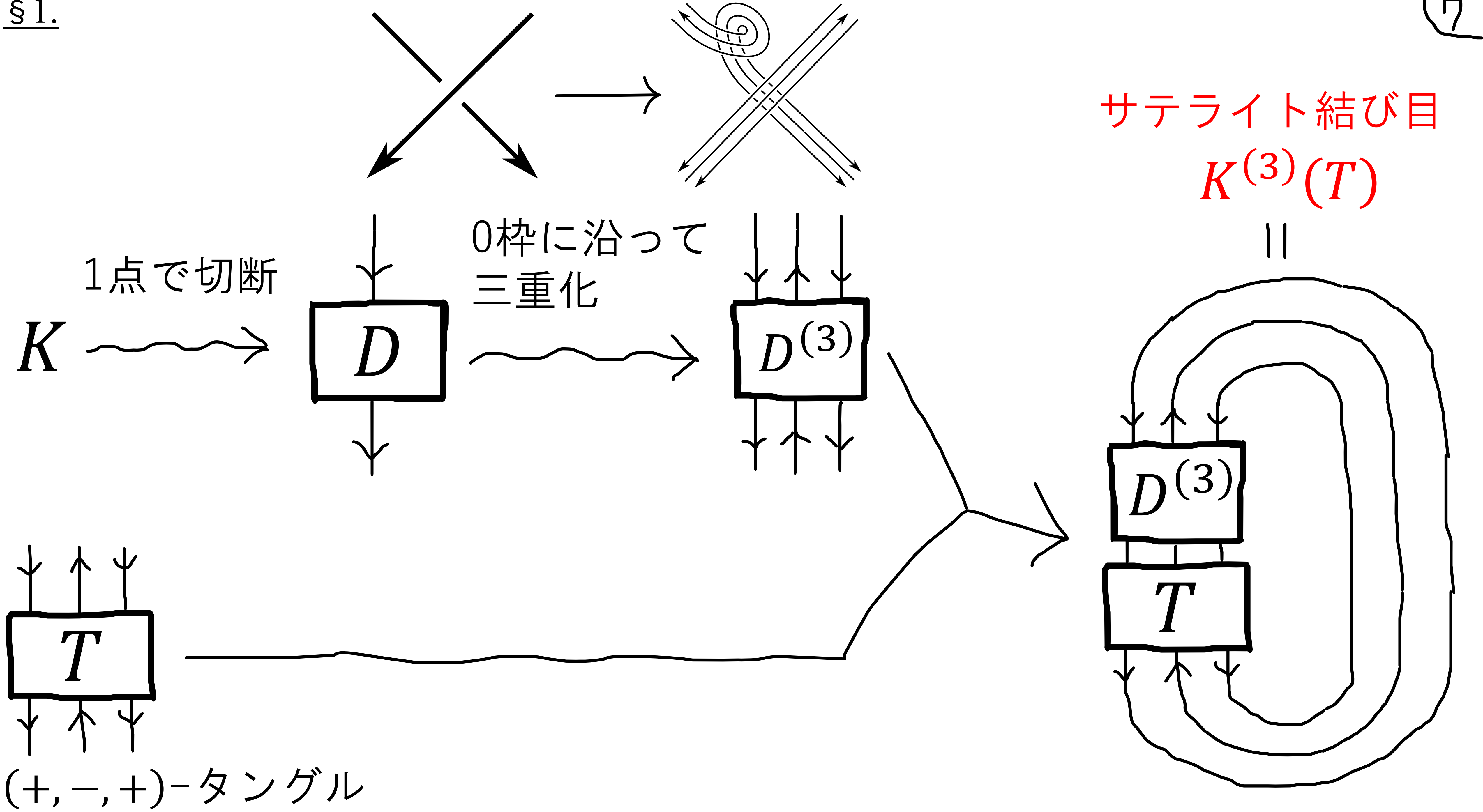
$W_\theta\Bigg($  $\Bigg) = \theta(w,x,y)$, $W_\theta\Bigg($  $\Bigg) = \theta(w,x,y)^{-1}$

$$\Psi_\theta(D,\mathcal{C}) = \prod_{\alpha:\,crossing\ of\ D} W_\theta(\alpha,\mathcal{C};\text{非有界領域のcolor}) \in A$$

<u>事実</u>　非有界領域のcolorによらない.

$$\Psi_\theta(D) = \sum_{\mathcal{C}:\,X\ coloring\ of\ D} \Psi_\theta(D,\mathcal{C}) \in \mathbb{Z}[A]$$　シャドーコサイクル不変量

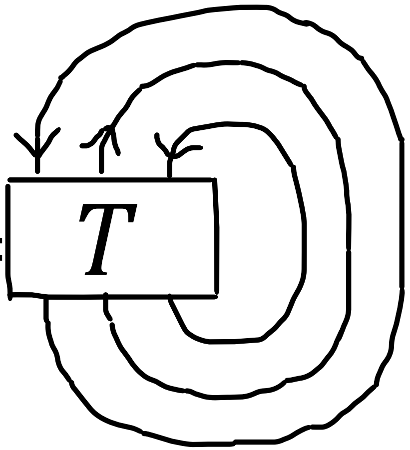
§1.



§2. 主結果

定理1　$\phi \in H^3(R_p; \mathbb{Z}_p) \cong \mathbb{Z}_p$　Mochizuki 3-コサイクル

$$\Psi_\phi\left(K^{(3)}(T)\right) = \frac{\Psi_\phi(K)\Psi_\phi(\bar{T})}{p} \in \mathbb{Z}[t]/(t^p - 1)$$

但し$\mathbb{Z}_p = \langle t \mid t^p = 1 \rangle$,　$\bar{T} =$ 

§2. 定理2

$$\begin{array}{ccl} H^3(R_3;\mathbb{Z}_3) & \longrightarrow & H^3(Q_6;\mathbb{Z}_3) \ni \phi_6 \\ \Cup & \searrow & \\ \phi & & H^3(Q_6';\mathbb{Z}_3) \ni \phi_6' \end{array}$$

$f: Q_6 \twoheadrightarrow R_3$

$Q_6' \twoheadrightarrow R_3$

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$$\begin{array}{ccccccc} Q_6 \times \overline{Q_6} \times Q_6 & \xrightarrow{\cong} & \bigsqcup_{i=1,\cdots,4} (Q_6)_i & \sqcup & \bigsqcup_{j=1,\cdots,8} (Q_{12})_j & \sqcup & \bigsqcup_{k=1,\cdots,8} (Q_{12}')_k \\ \Cup & & \Cup & & \Cup & & \Cup \\ \mathbf{x}_i, \mathbf{y}_j, \mathbf{z}_k, \widetilde{\mathbf{y}_j} & & x_i & & y_j,\ \widetilde{y_j} & & z_k \end{array}$$

$$\Psi_{\phi_6}(K^{(3)}(T)) = \sum_{i=1,\cdots,4} \Psi_{\phi_6}(T;\mathbf{x}_i,\mathbf{x}_i)\Psi_{\phi_6}(K)$$

$$+4\sum_{j=1,\cdots,4} \Psi_{\phi_6}(T;\mathbf{y}_j,\mathbf{y}_j)\cdot \sum_{\Phi_\alpha(K,\mathcal{C})=1} \Psi_{\phi_6}(K,\mathcal{C})$$

$$+4\sum_{j=1,\cdots,4} \Psi_{\phi_6}(T;\mathbf{y}_j,\widetilde{\mathbf{y}_j})\cdot \sum_{\Phi_\alpha(K,\mathcal{C})=s} \Psi_{\phi_6}(K,\mathcal{C})$$

$$+4\sum_{k=1,\cdots,4} \Psi_{\phi_6}(T;\mathbf{z}_k,\mathbf{z}_k)\Psi_{\phi_6'}(K) \in \mathbb{Z}[t]/(t^3-1)$$

$\alpha: Q_6 \times Q_6 \to \mathbb{Z}_2$

2−コサイクル

$\updownarrow$

$Q_{12} \to Q_6$

中心拡大

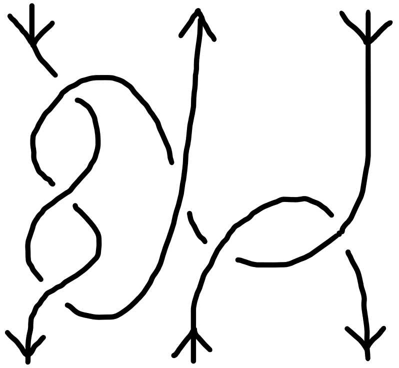
§2. 定理3

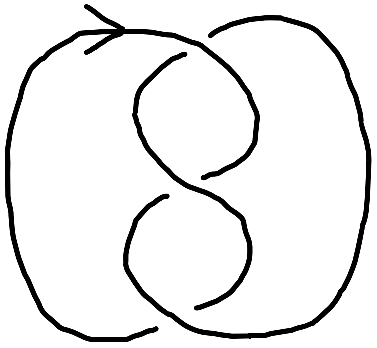
$$\begin{array}{ccc} H^3(R_3;\mathbb{Z}_3) & \to & H^3(Q_6;\mathbb{Z}_3) \\ \rotatebox[origin=c]{90}{$\in$} & & \rotatebox[origin=c]{90}{$\in$} \\ \phi & & \phi_6 \end{array}$$

$$\begin{tikzcd}
 & \gamma_1 \quad \gamma_2 \quad \gamma_3 \arrow[d] & \gamma \arrow[d] \arrow[ddr, mapsto, bend left] & \\
Q(K^{(3)}(T)) \arrow[r, phantom, "\supset"] \arrow[d, "\mathcal{C}"'] & \{D^{(3)}\text{の}arc\} \arrow[d, "\mathcal{C}'"] & \{D\text{の}arc\} \arrow[d, "\mathcal{C}^{(3)}"] & \\
Q_6 \arrow[r, "f"'] & R_3 & R_3 & \\
 & & & \mathcal{C}'(\gamma_1) - \mathcal{C}'(\gamma_2) + \mathcal{C}'(\gamma_3)
\end{tikzcd}$$

$$\Psi_{\phi_6}\left(K^{(3)}(T), \mathcal{C}\right) = \Psi_{\phi}\left(K, \mathcal{C}^{(3)}\right)\Psi_{\phi}(\bar{T}, f\mathcal{C}|_T) \in \mathbb{Z}_3$$

例(定理1　2面体カンドルR_3)

$T_1 =$ 

$K =$ 

$\Psi_\phi(\overline{T}_1) = 3(1+2t^2)$

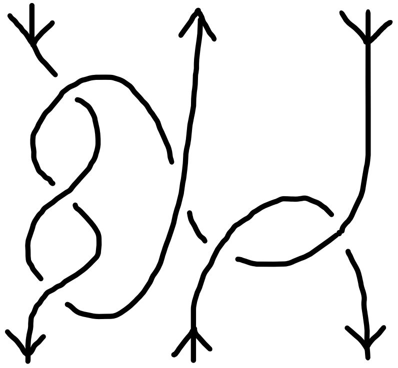
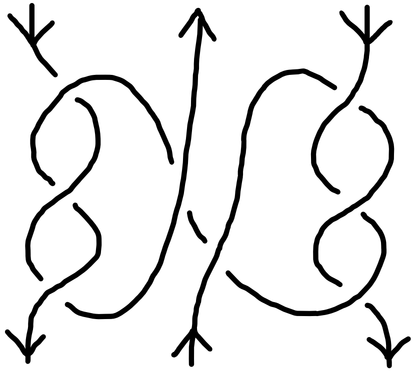
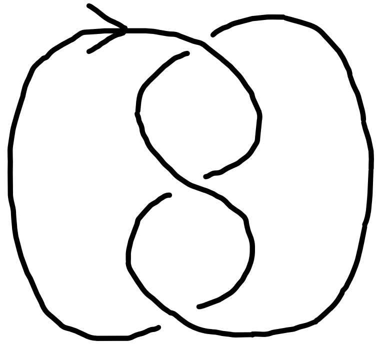
$\Psi_\phi(K) = 3(1+2t)$

定理1

$$\Psi_\phi\left(K^{(3)}(T_1)\right) = \frac{\Psi_\phi(\overline{T}_1)\Psi_\phi(K)}{3} = 3(1+2t^2)(1+2t)$$

$$= 3(5+2t+2t^2) \in \mathbb{Z}[t]/(t^3-1)$$

§2. 例(定理2　8面体カンドルQ_6)

$T_1 =$  $T_2 =$  $K =$ 

$$\Psi_{\phi_6}(K^{(3)}(T_1)) = (1+4t^2)\Psi_{\phi_6}(K) = (1+4t^2)6(1+4t)$$

定理2

$$= 6(17+4t+4t^2) \in \mathbb{Z}[t]/(t^3-1)$$

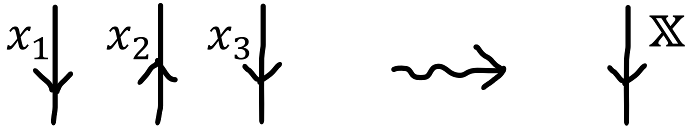
$$\Psi_{\phi_6}(K^{(3)}(T_2)) = (1+8t^2)\underbrace{\Psi_{\phi_6}(K)}_{6(1+4t)} + 8t\underbrace{\Psi_{\phi_6'}(K)}_{6(1+4t)} + 8t\underbrace{\sum_{\Phi_\alpha(K,\mathcal{C})=1} \Psi_{\phi_6}(K,\mathcal{C})}_{6}$$

定理2

$$= 6(33+20t+40t^2) \in \mathbb{Z}[t]/(t^3-1)$$

§3. 証明の概要

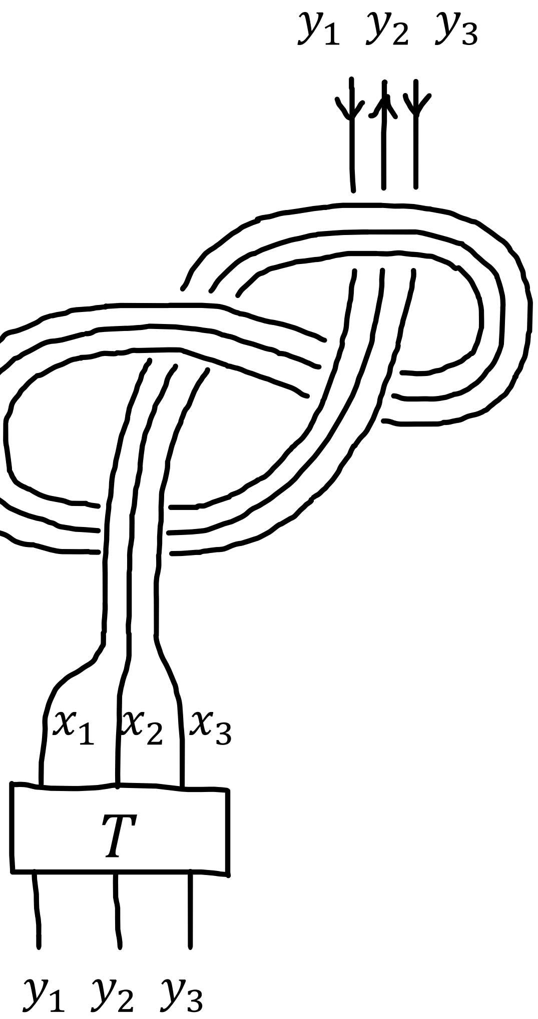
方針

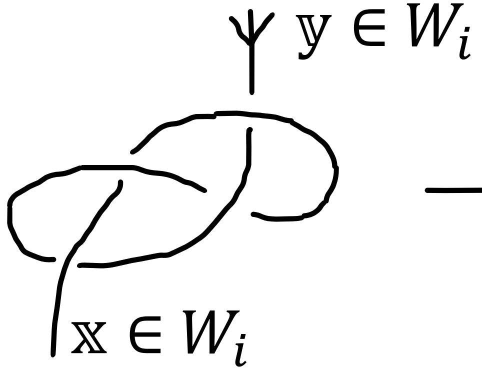


$(x_1, x_2, x_3) = \mathbb{x}$

$X \times \overline{X} \times X \cong \bigsqcup_i W_i$ 連結なカンドルへの分解

ψ_i W_iのコサイクル





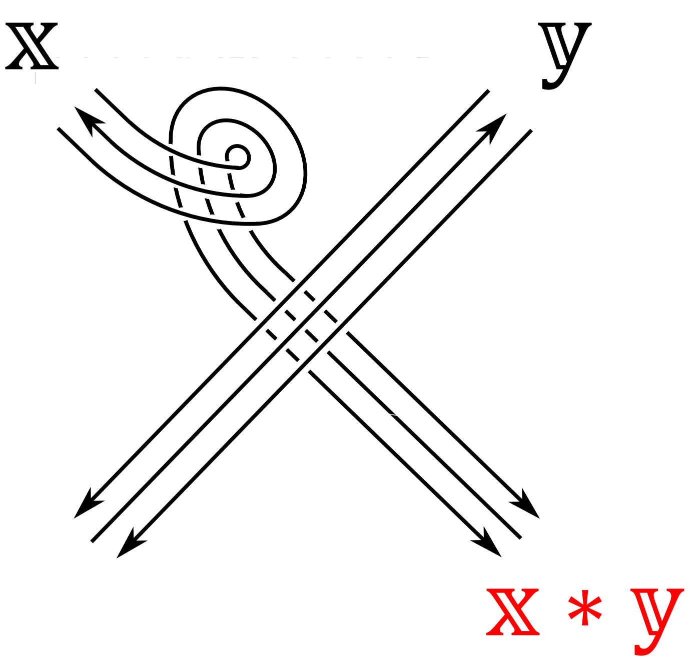
$\longrightarrow \Psi_{\psi_i}(D; \mathbb{y}, \mathbb{x})$

$\Psi_\theta(T; \mathbb{x}, \mathbb{y})$

$\Psi_\theta\left(K^{(3)}(T), \mathcal{C}\right)$

§3. $X^{(3)} = X \times \bar{X} \times X,\ *: X^{(3)} \times X^{(3)} \to X^{(3)}$

$\mathbb{x}, \mathbb{y} \in X^{(3)}$に対し$\mathbb{x} * \mathbb{y} \in X^{(3)}$



命題　$(X^{(3)}, *)$はカンドル.

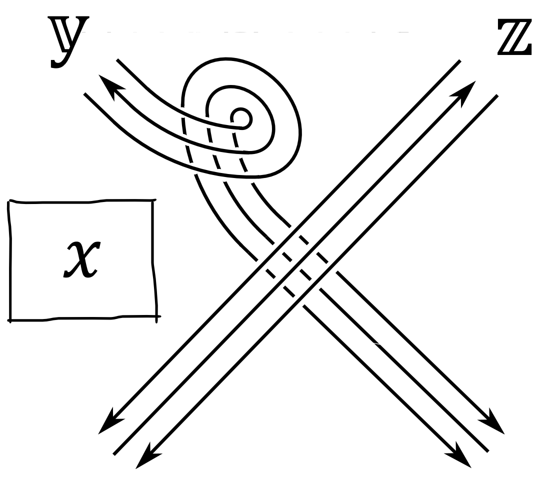
$\theta: X^3 \to A$　3-コサイクル

$$X^{(3)} \cong \sqcup_i W_i$$

$\psi_i: X \times W_i \times W_i \to A$

シャドーカンドル2-コサイクル

$\psi_i(x, \mathbb{y}, \mathbb{z}) = W_\theta$ (



)

§3.

R_pの場合

$$\left(R_p\right)^{(3)} = \bigsqcup_{i,j\in\mathbb{Z}_p}\left(R_p\right)_{i,j} \qquad \left(R_p\right)_{i,j} = \{(x+i\,,x+i+j\,,x+j) \mid x \in R_p\} \cong R_p$$

Q_6の場合

$$\begin{array}{l} (Q_6)^{(3)} = 4Q_6 \sqcup 8Q_{12} \sqcup 8Q'_{12} \\ \quad \Big\downarrow f \\ (R_3)^{(3)} = \bigsqcup_{i,j\in\mathbb{Z}_3}(R_3)_{i,j} \end{array}$$

$$f^{-1}\left((R_3)_{i,j}\right) = \begin{cases} 4Q_6 & (i,j) = (0,0), \\ Q_{12} \sqcup Q'_{12} & (i,j) \neq (0,0), \end{cases}$$

今後の課題

- Q_6のϕ_6以外のコサイクルに対しサテライト化公式はどのような形で書けるか. （定理2の拡張）

- どのような(X,θ)に対し$\Psi_\theta\left(K^{(3)}(T)\right)$は積で表されるか.(定理1の一般化)

- KとTのトポロジー $\rightsquigarrow$ サテライト化公式の形

どのように影響するか？