Strong and weak (1, 2, 3) homotopies on knot projection

伊藤昇氏(早稲田大学)との共同研究

学習院中等科
瀧村祐介

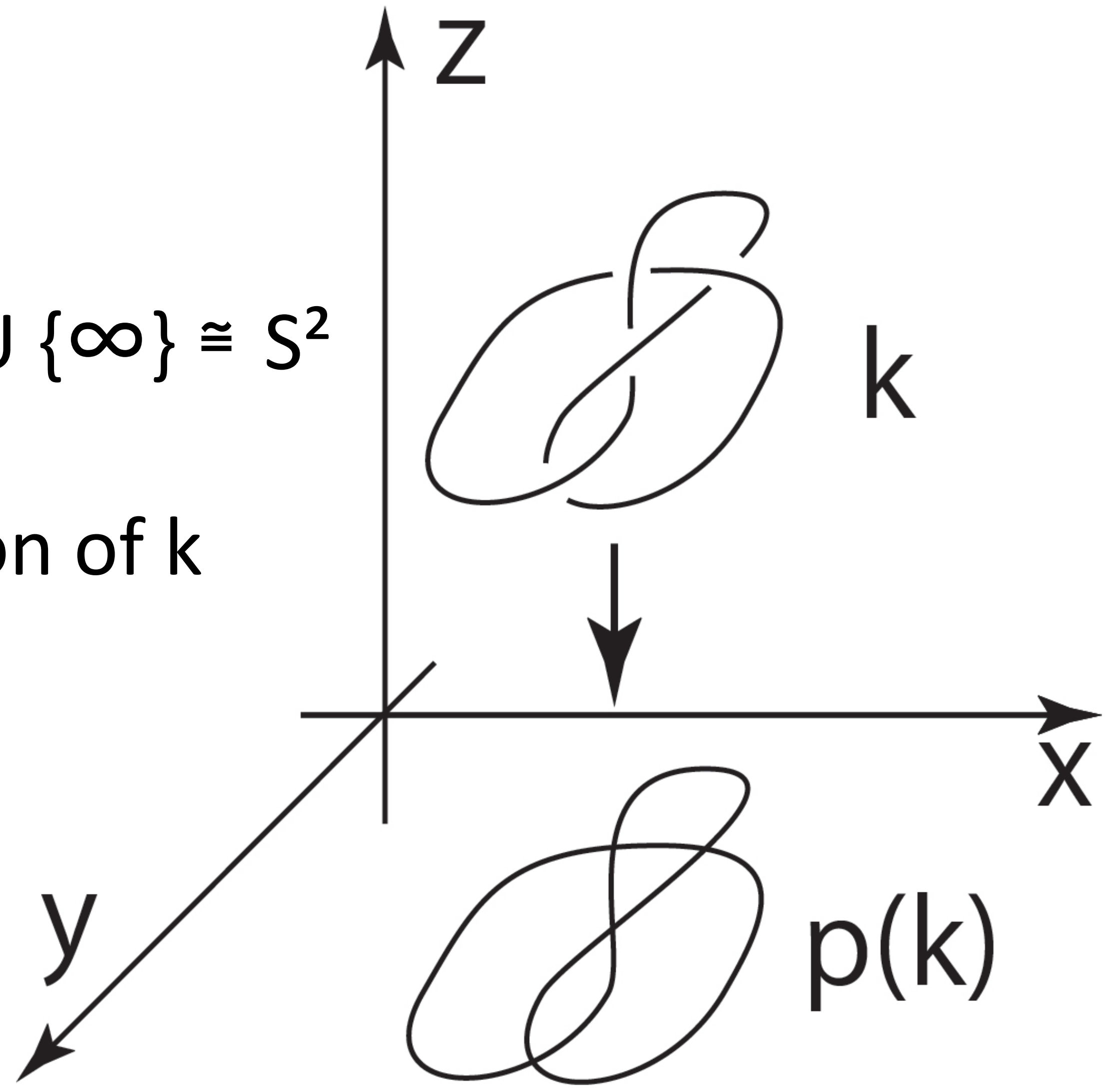
Definition

k: knot in R^3

p : $R^3 \longrightarrow R^2 \subset R^2 \cup \{\infty\} \cong S^2$

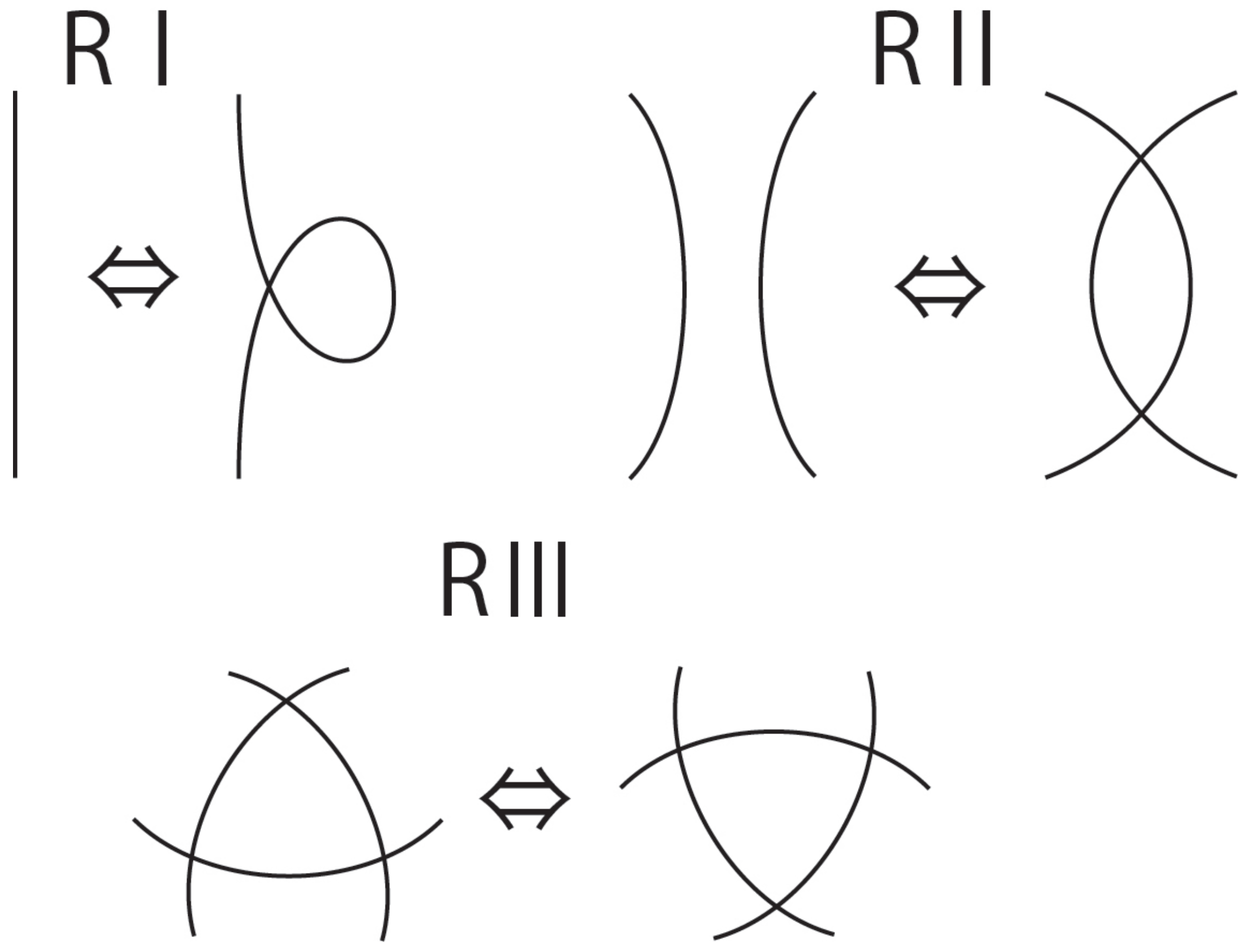
p(k) : knot projection of k

以後、P と表す



Definition

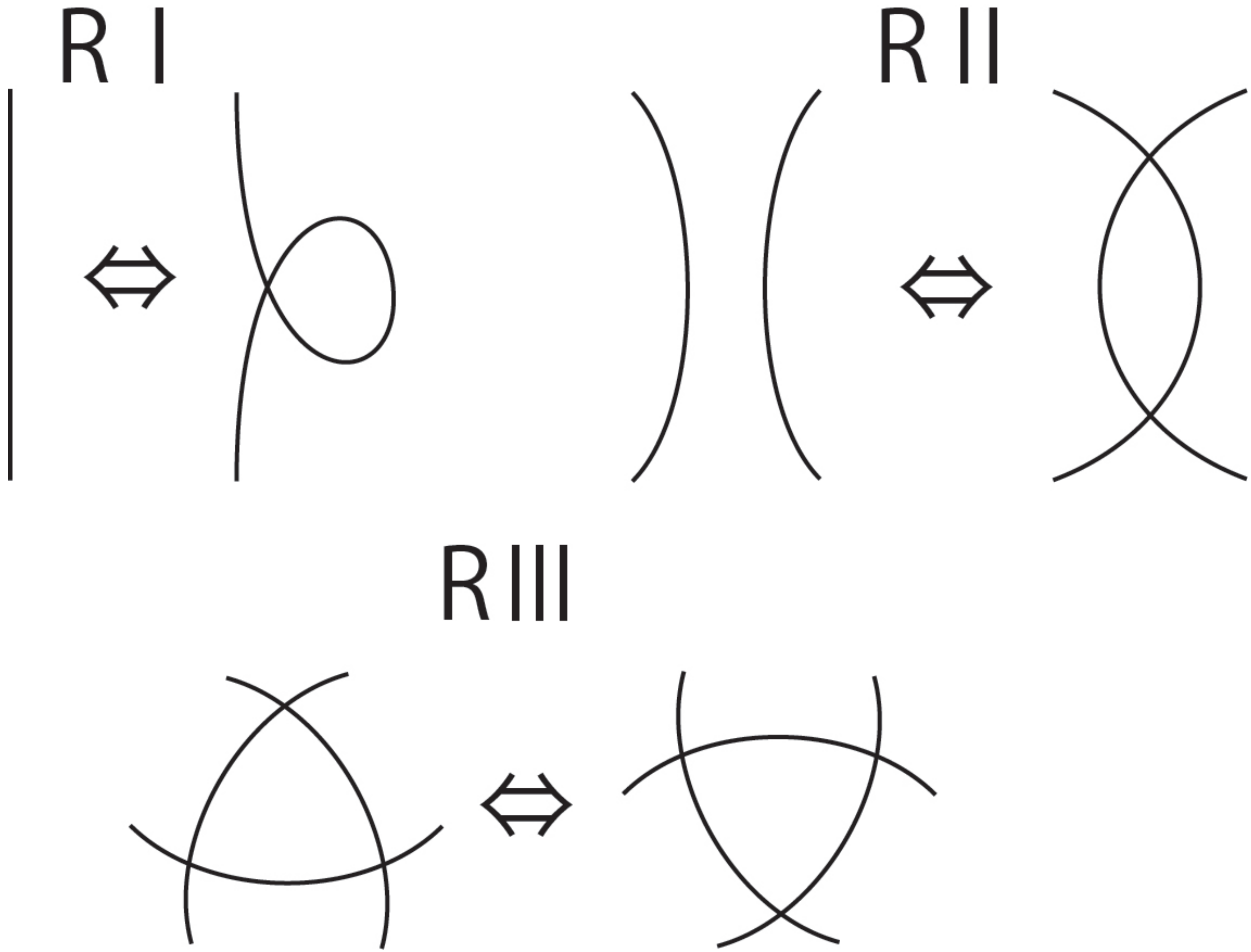
Reidemeister moves on knot projection

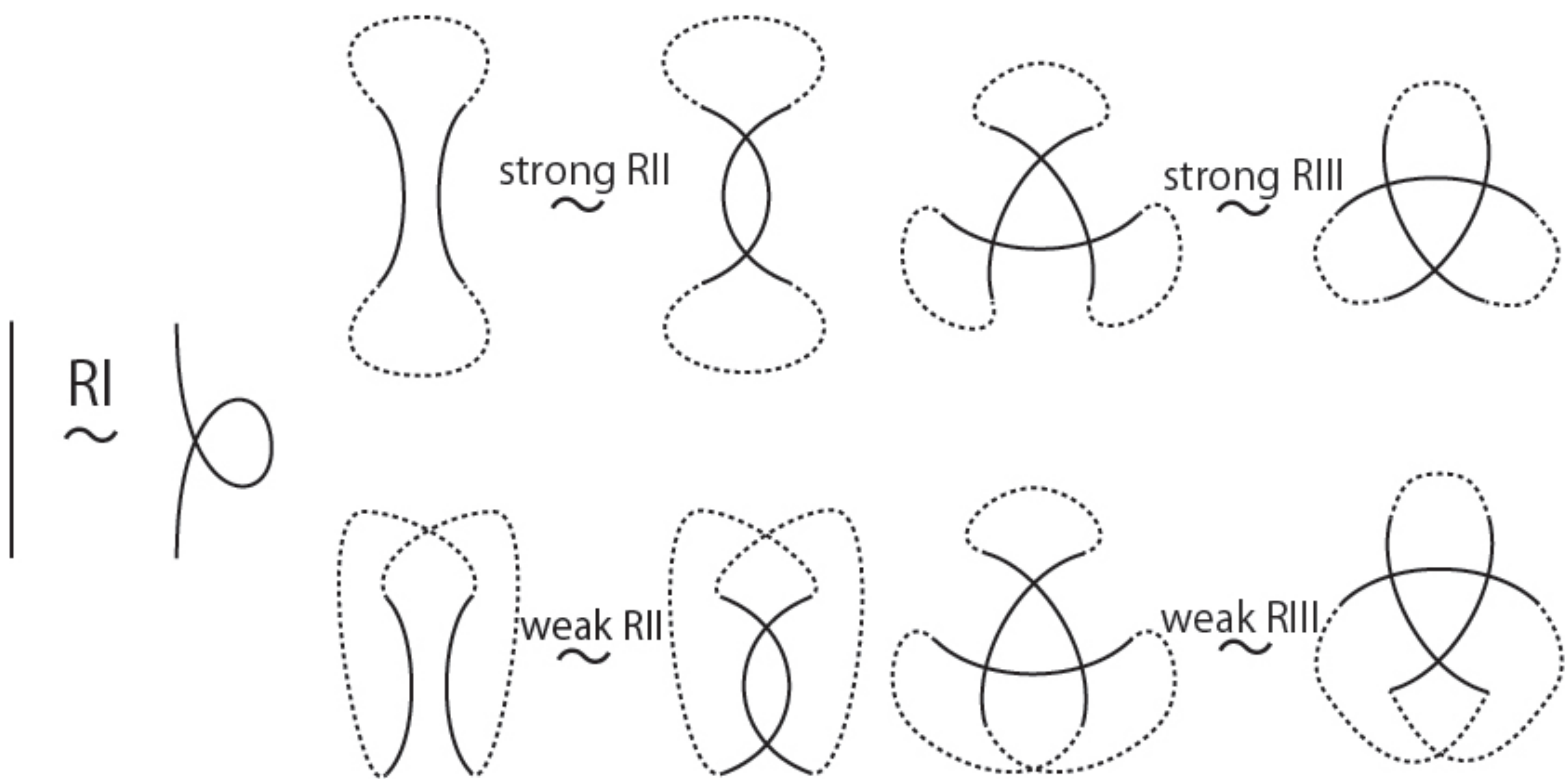


Proposition

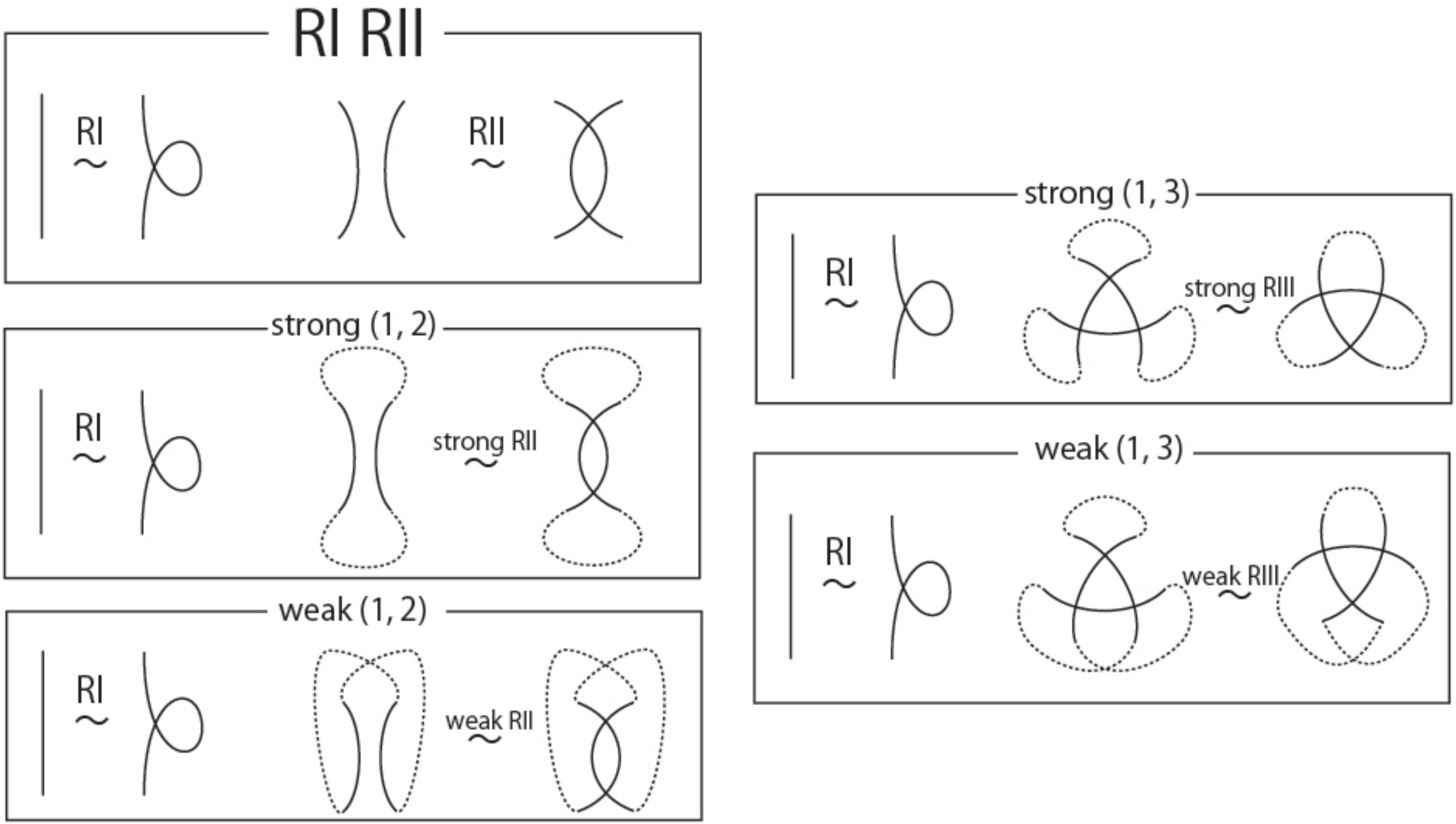
∀ P_1 , P_2 : knot projections on S^2

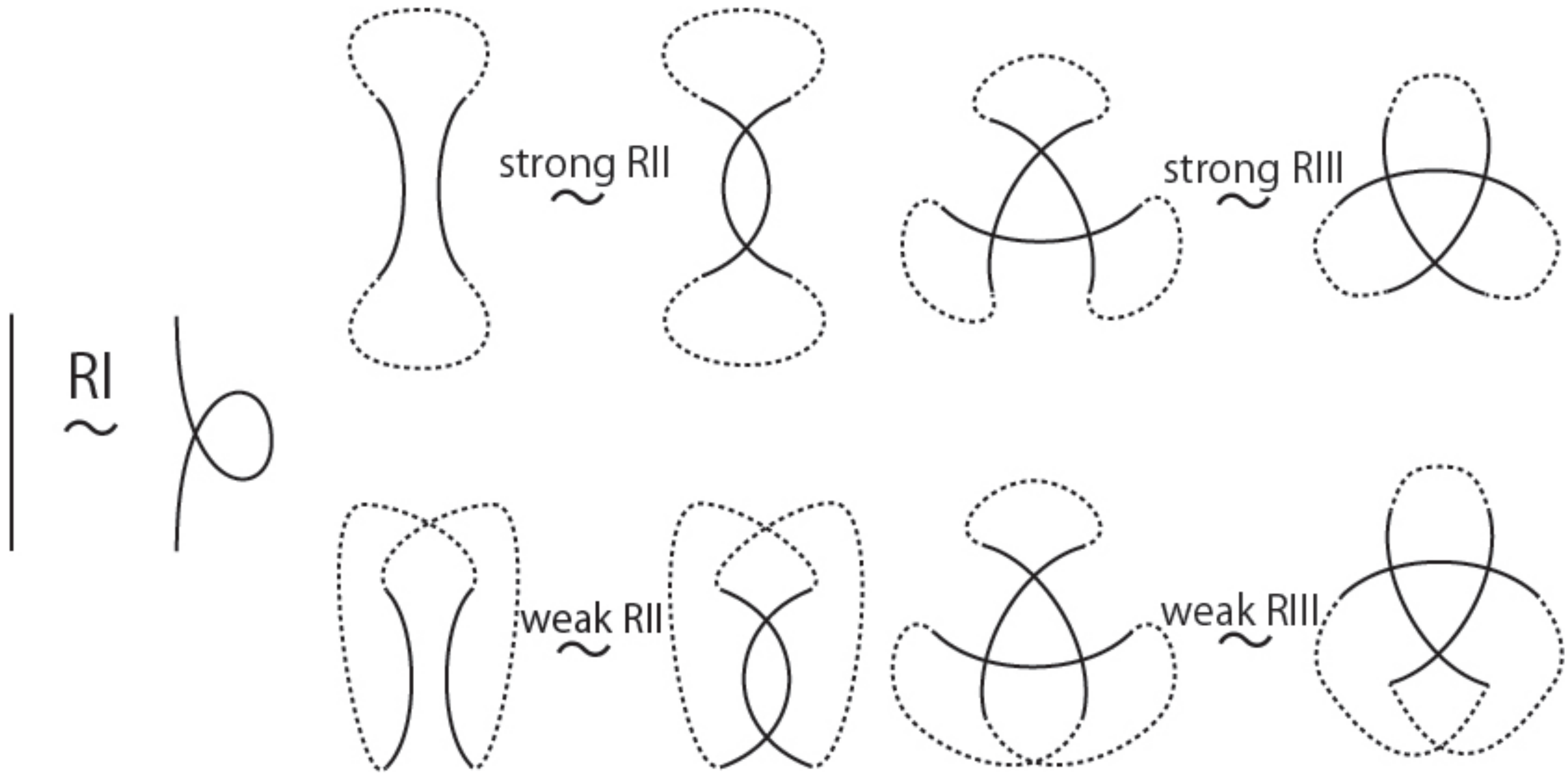
P_1 , P_2 は RI, RII, RIII で移り合う

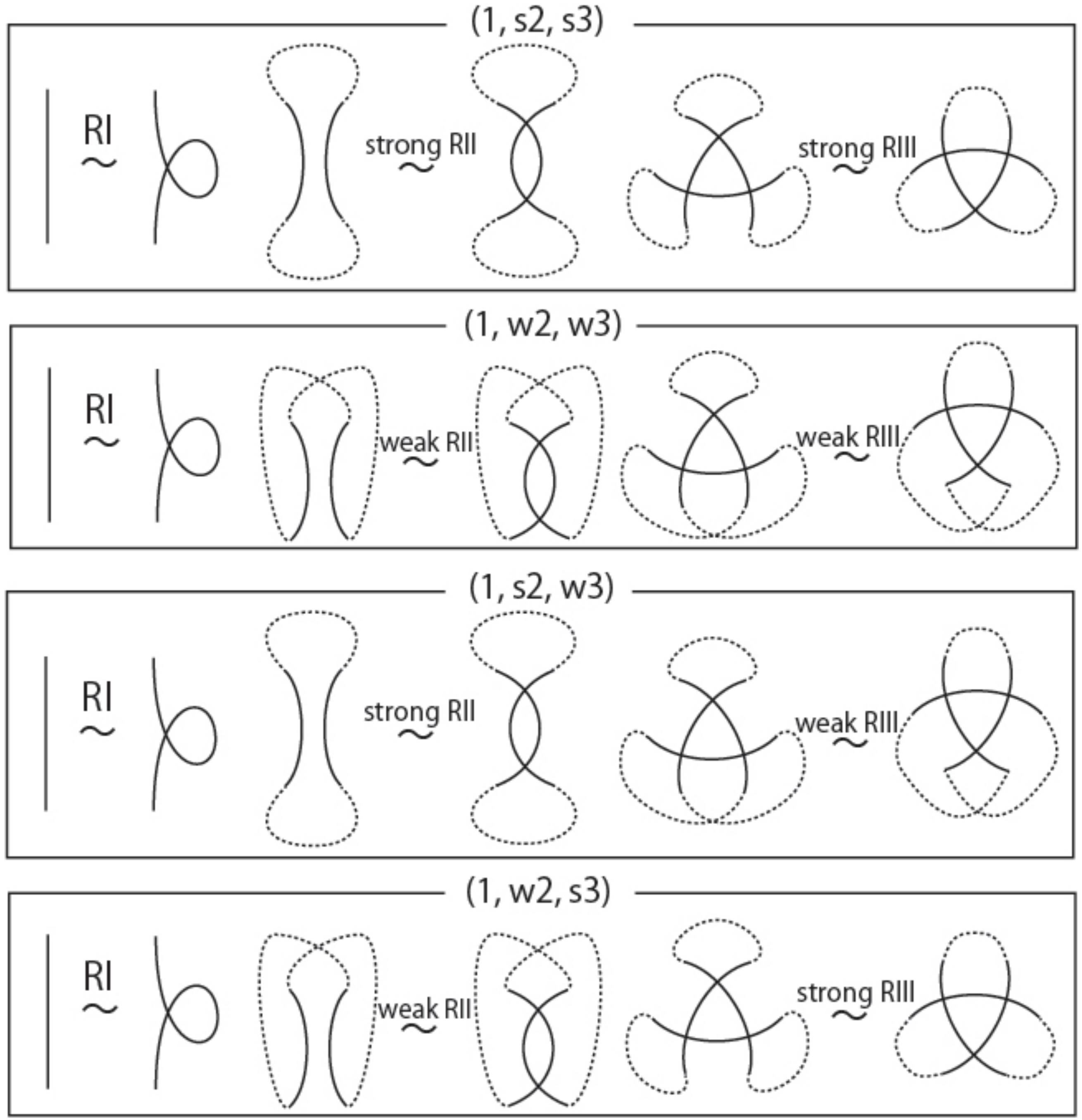

R I
R II
R III

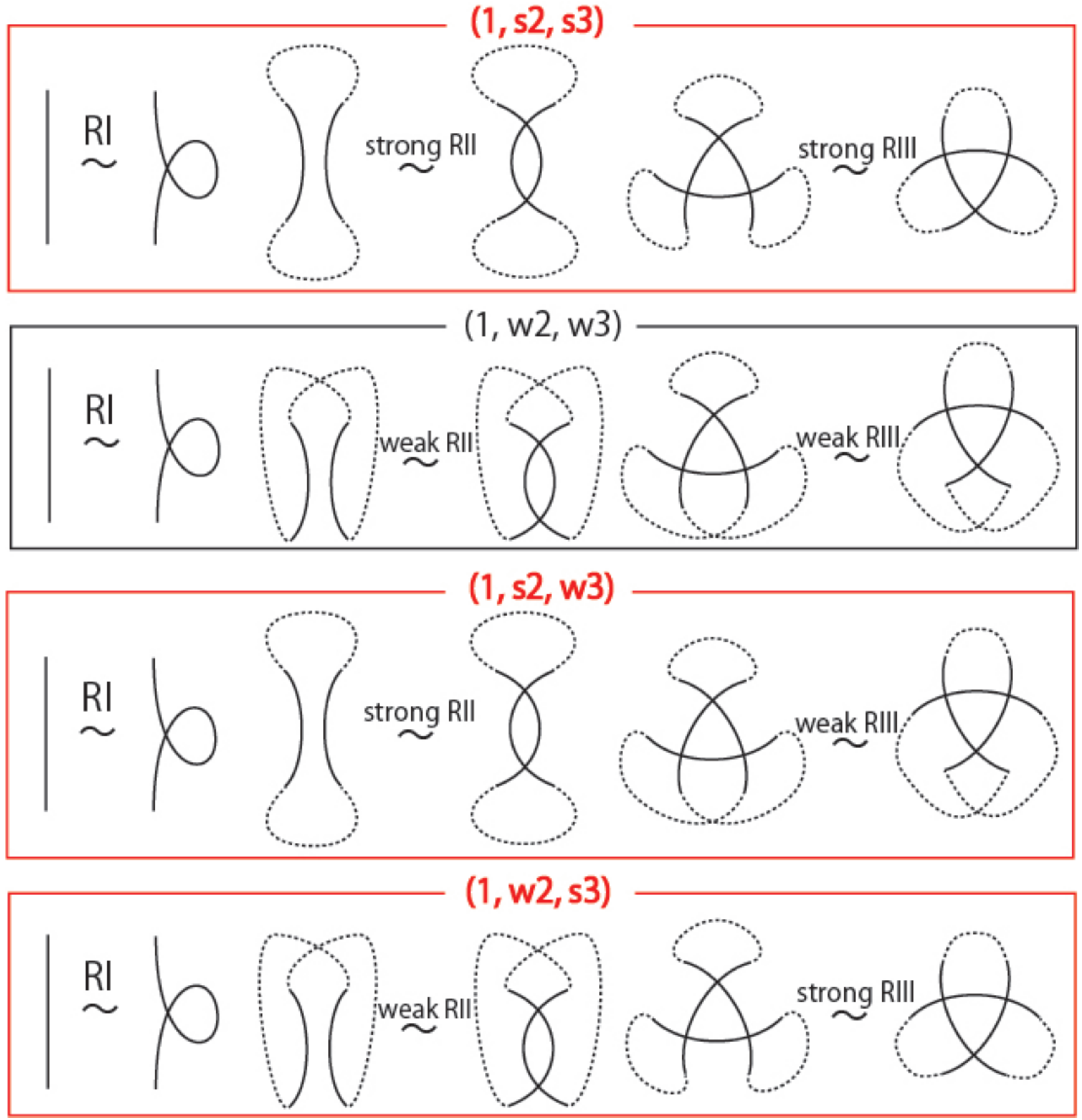

RI
~
strong RII
~
strong RIII
~
weak RII
~
weak RIII
~

今までの研究

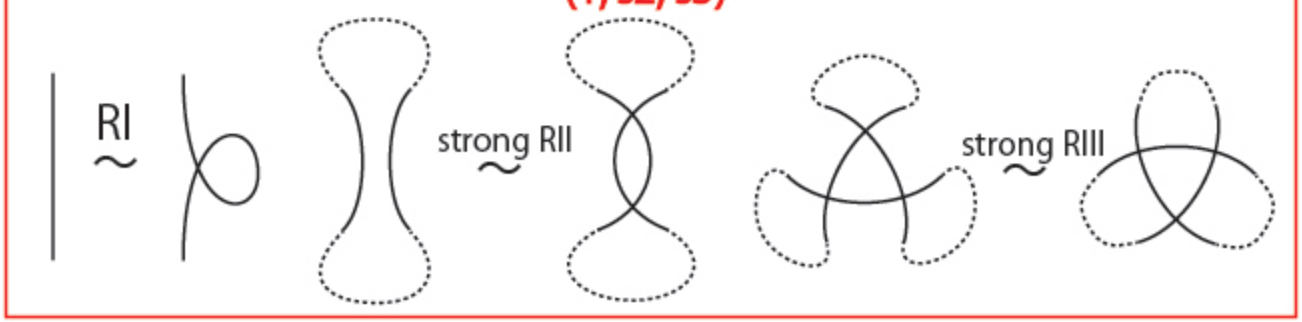

RI RII
RI
RII
strong (1, 2)
RI
strong RII
weak (1, 2)
RI
weak RII
strong (1, 3)
RI
strong RIII
weak (1, 3)
RI
weak RIII


RI
strong RII
strong RIII
weak RII
weak RIII

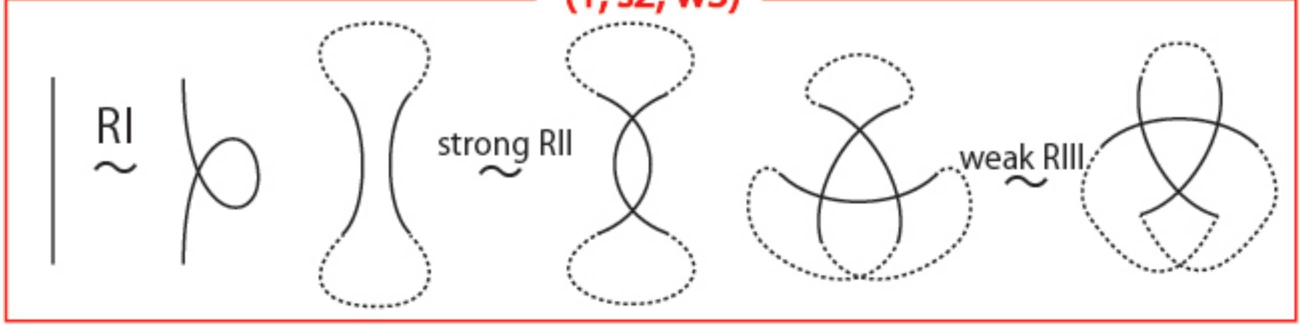

(1, s2, s3)
RI
strong RII
strong RIII
(1, w2, w3)
RI
weak RII
weak RIII
(1, s2, w3)
RI
strong RII
weak RIII
(1, w2, s3)
RI
weak RII
strong RIII


(1, s2, s3)
RI
strong RII
strong RIII
(1, w2, w3)
RI
weak RII
weak RIII
(1, s2, w3)
RI
strong RII
weak RIII
(1, w2, s3)
RI
weak RII
strong RIII

(1, s2, s3)



(1, s2, w3)



(1, w2, s3)

Proposition

P_1 , P_2 : knot projections on S^2

(1) $^\forall P_1, P_2$ は (1, s2, s3) で移り合う

(2) $^\forall P_1, P_2$ は (1, s2, w3) で移り合う

(3) $^\forall P_1, P_2$ は (1, w2, s3) で移り合う

Proof of proposition

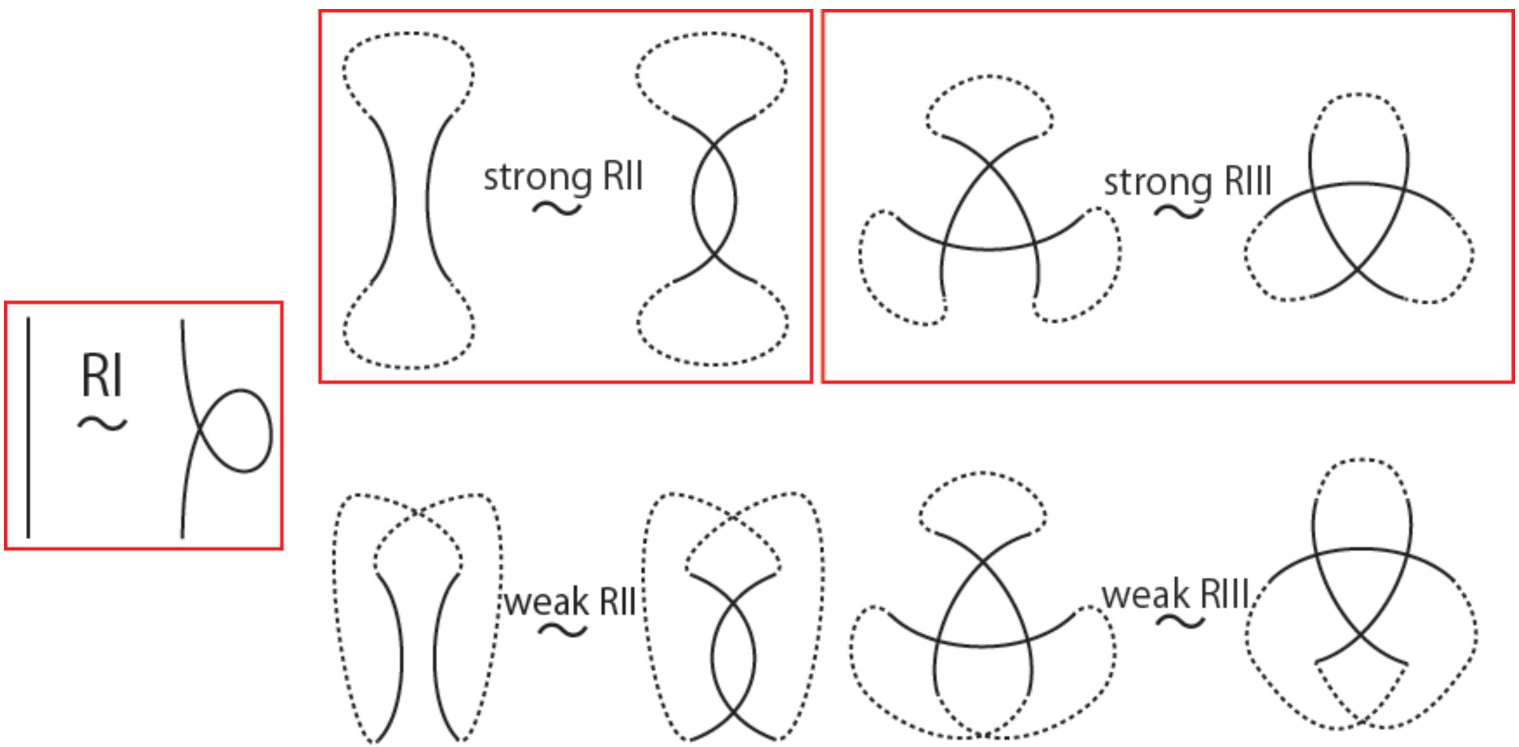
P_1 , P_2 : knot projections on S^2

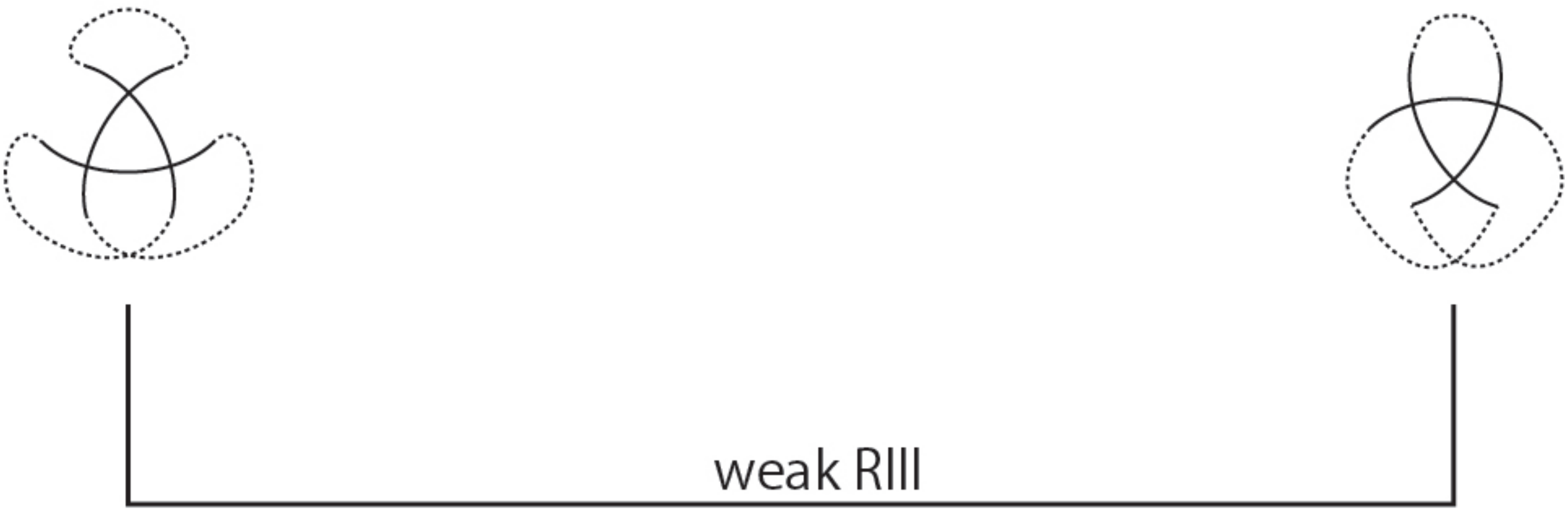
(1) $^\forall P_1, P_2$ は (1, s2, s3) で移り合う

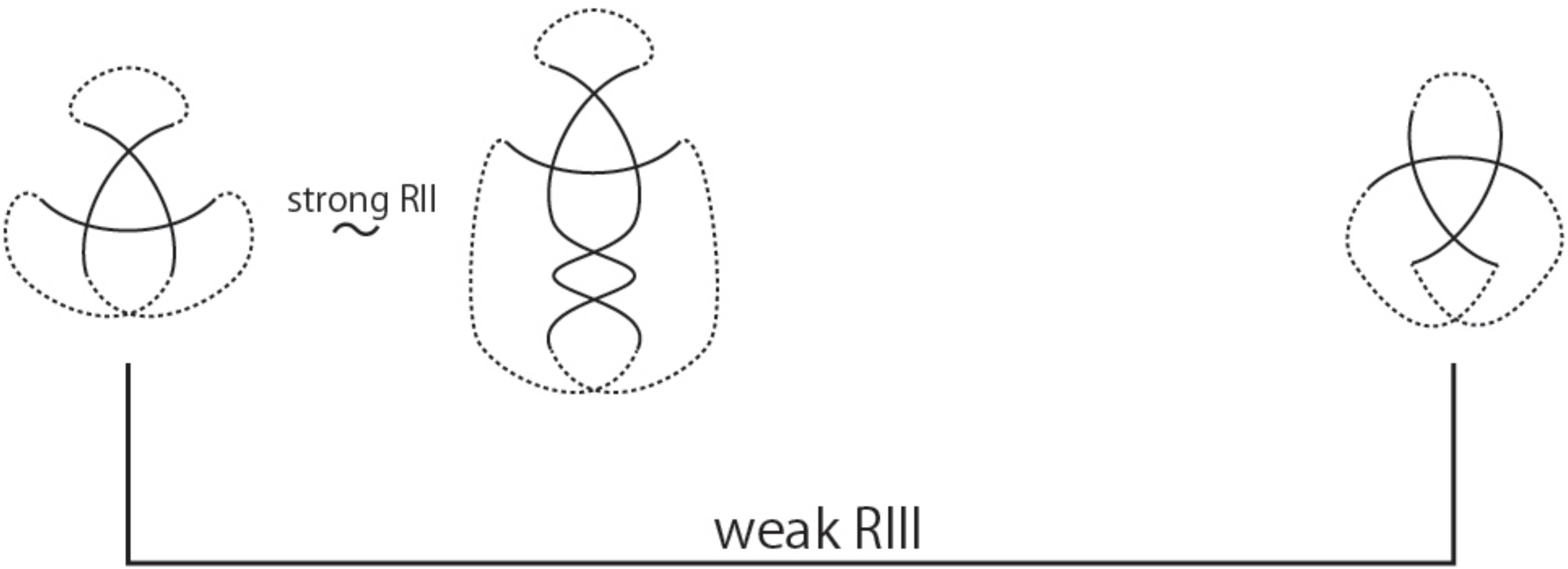
Proof of proposition

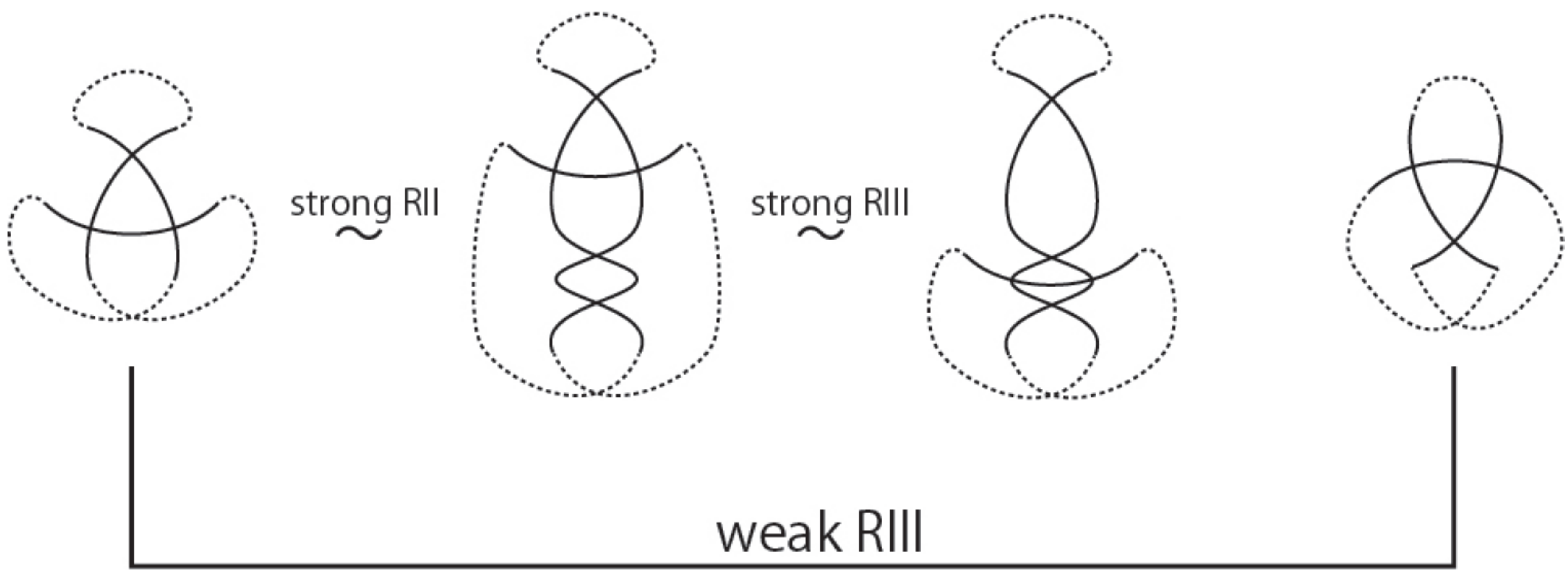
P_1 , P_2 : knot projections on S^2

(1) $^\forall P_1, P_2$ は (1, s2, s3) で移り合う

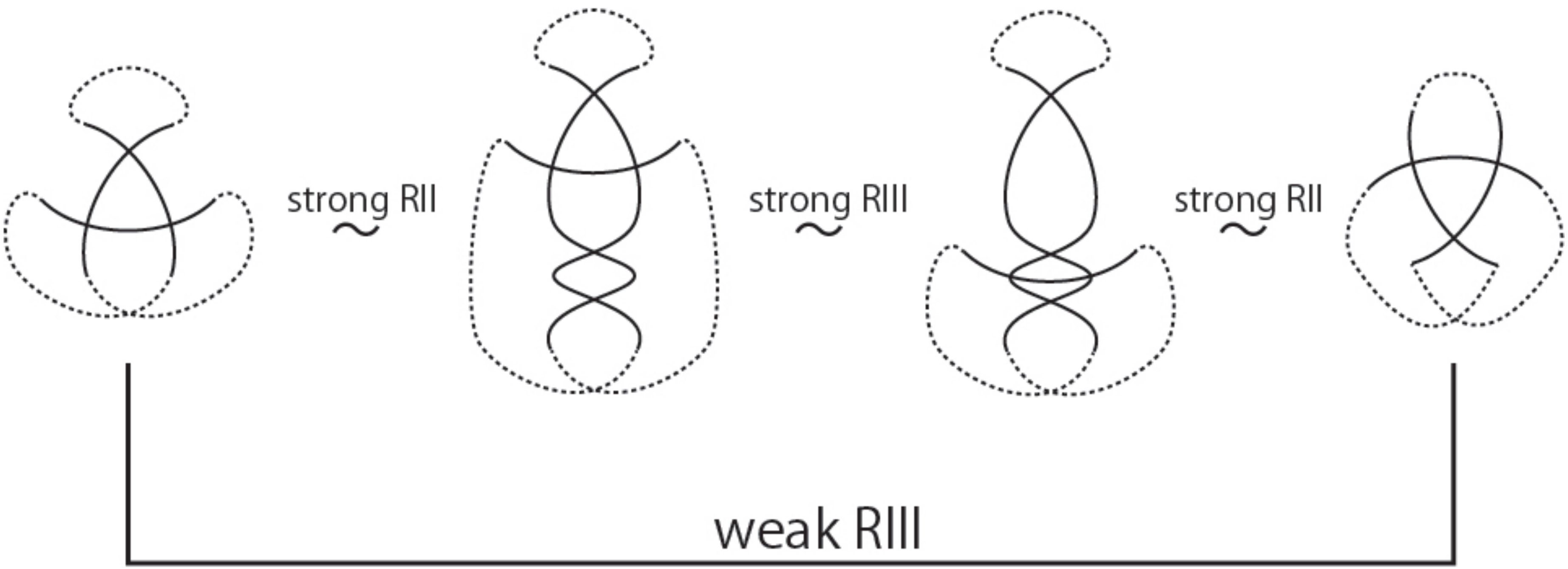


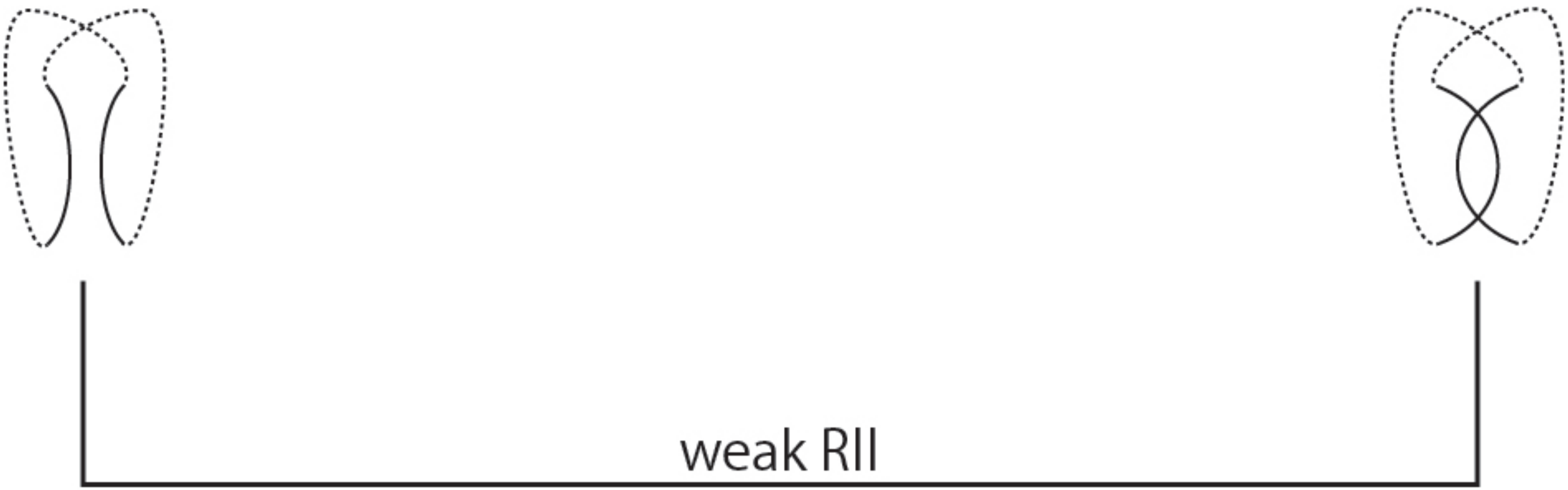

weak RIII

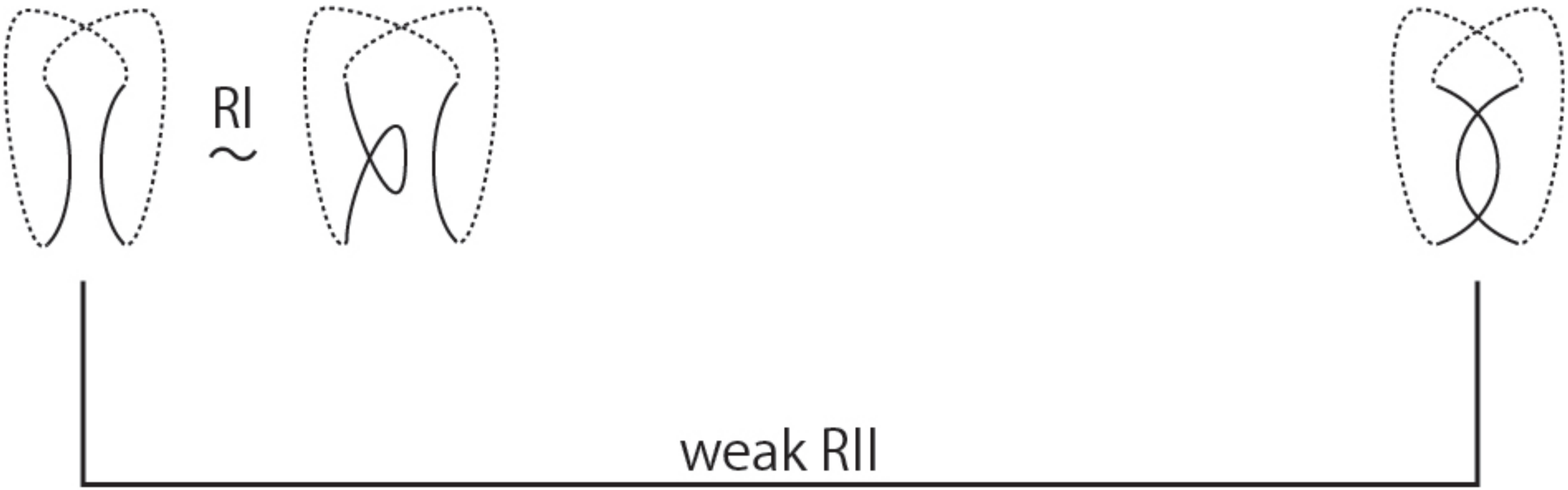

strong RII
~
weak RIII

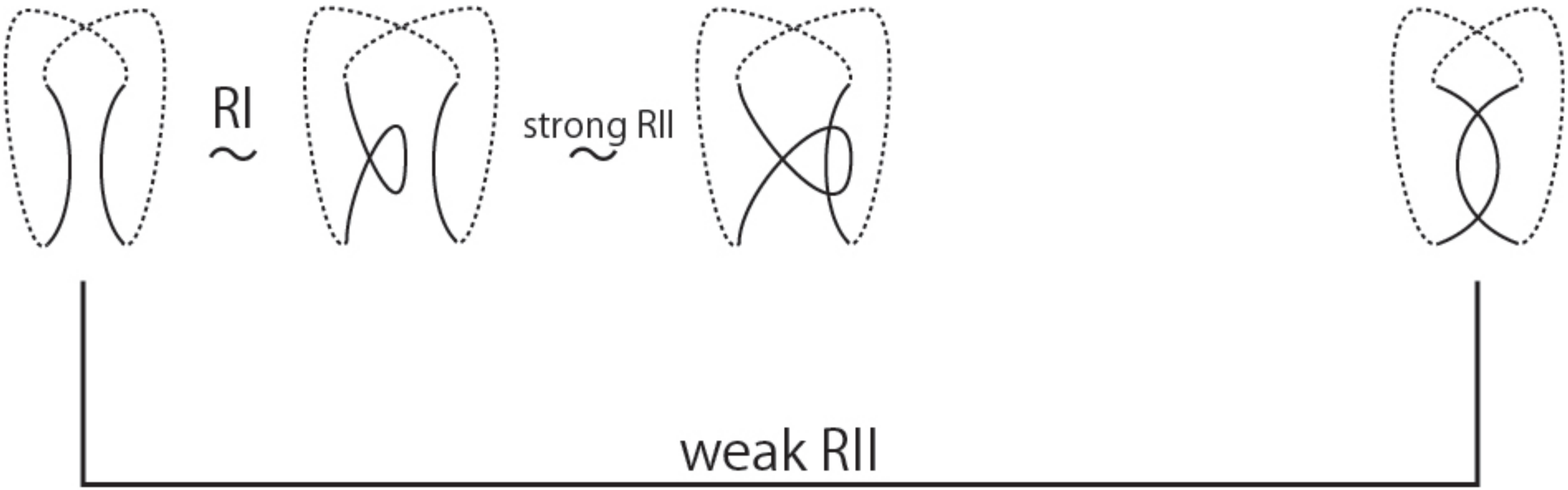


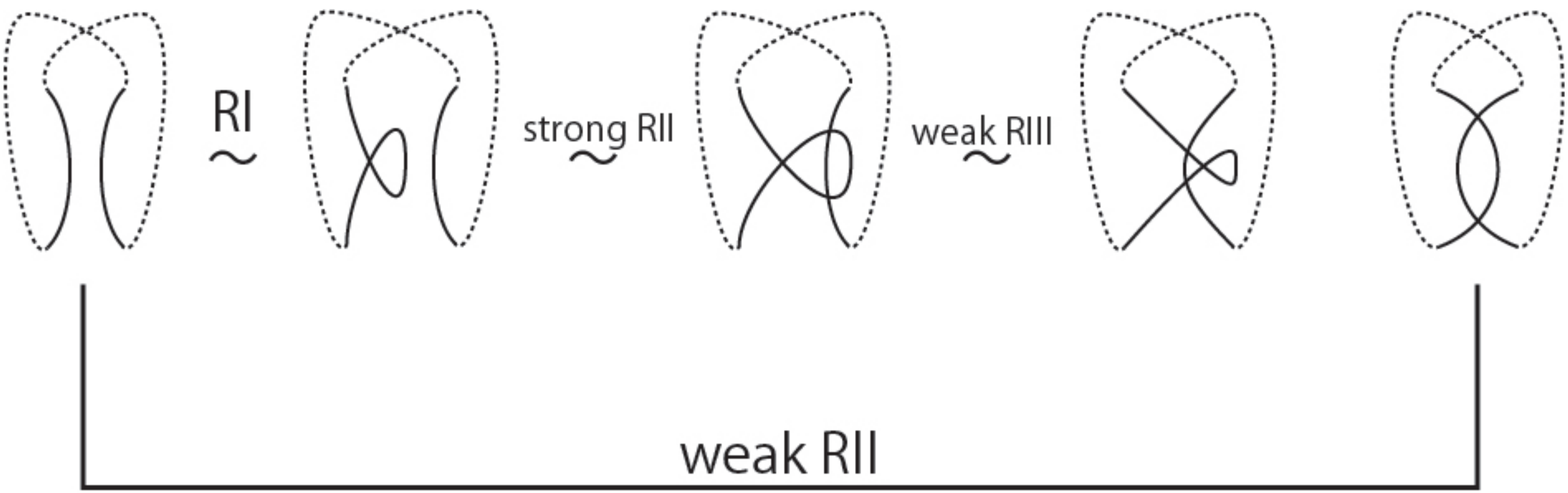
strong RII
~
strong RIII
~
weak RIII

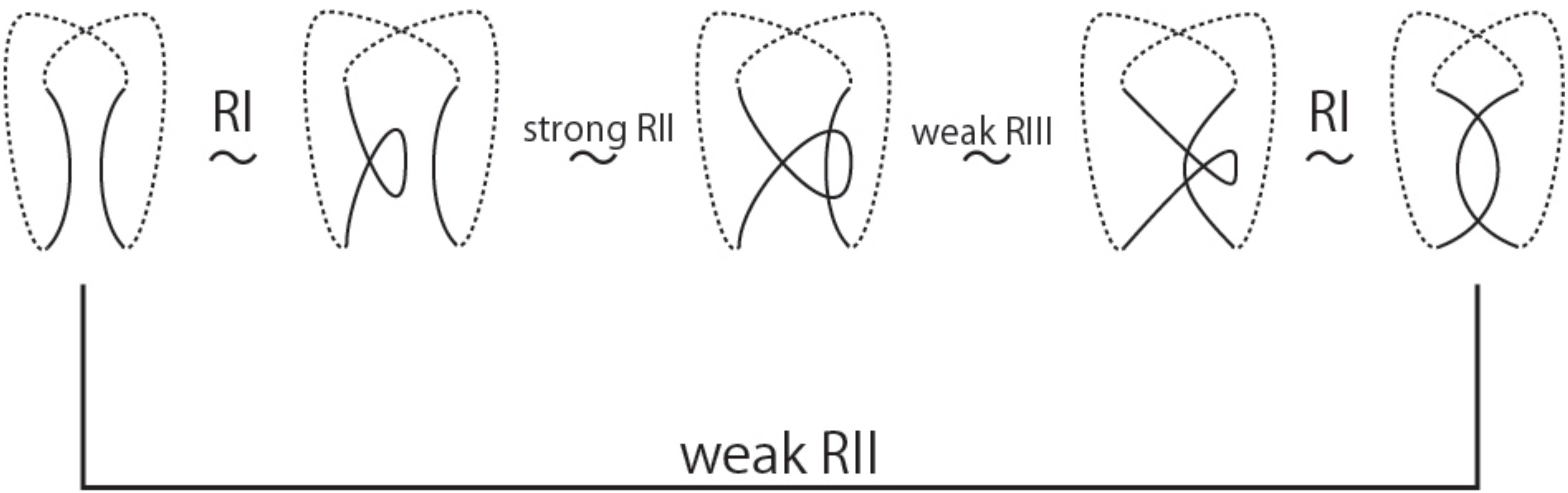

strong RII
~
strong RIII
~
strong RII
~
weak RIII


weak RII


RI
~
weak RII


RI
strong RII
weak RII


RI
strong RII
weak RIII
weak RII


RI
strong RII
weak RIII
RI
weak RII

Proposition

P_1 , P_2 : knot projections on S^2

(1) $^{\forall}P_1, P_2$ は (1, s2, s3) で移り合う

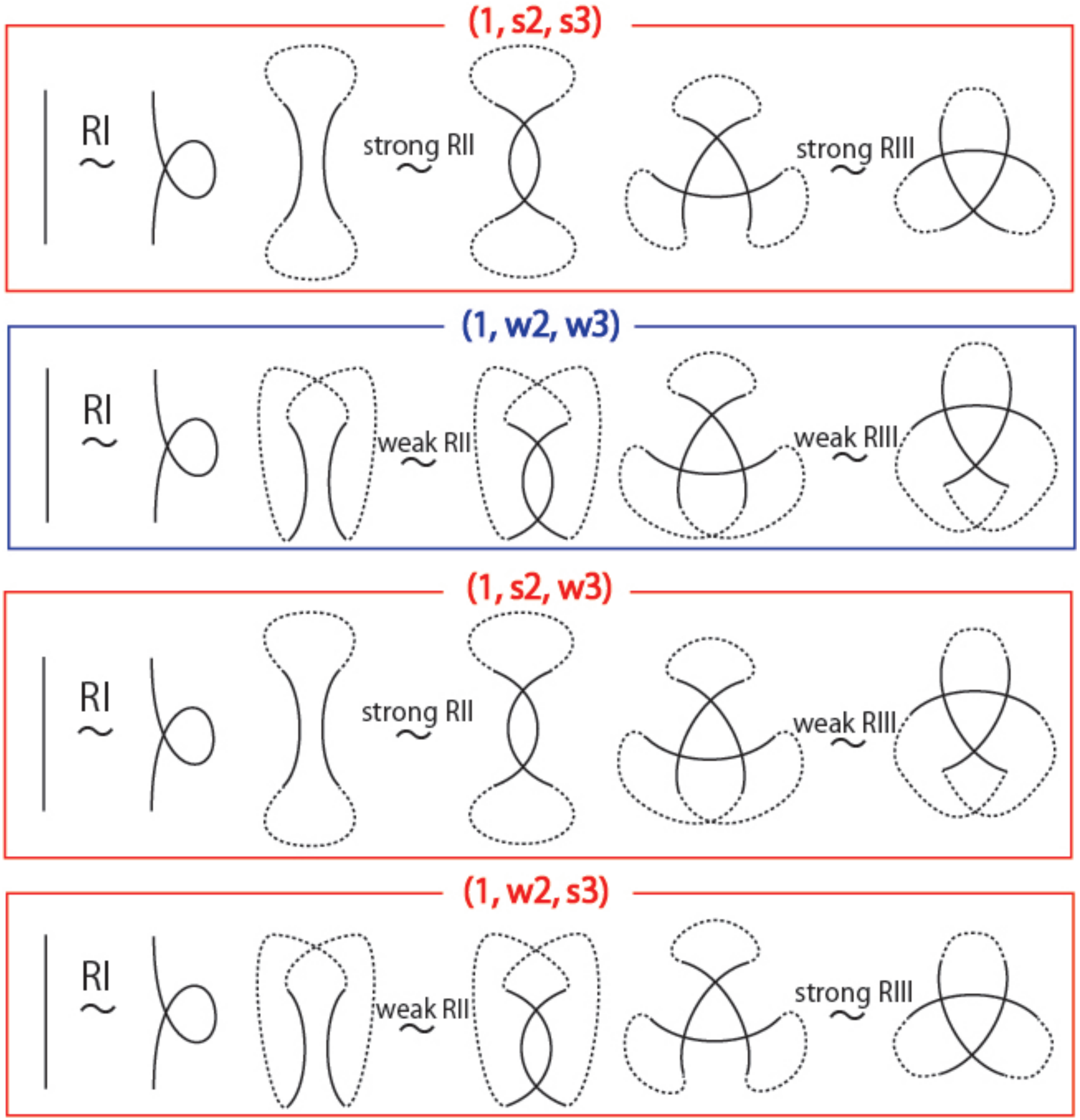
Proposition

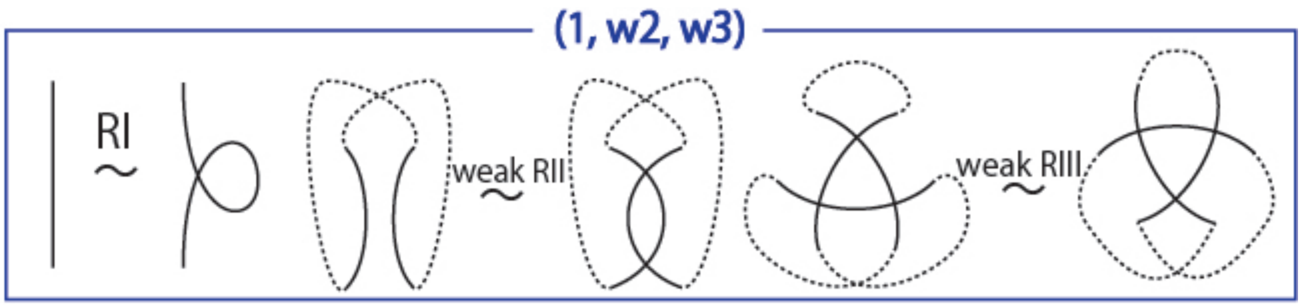
P_1 , P_2 : knot projections on S^2

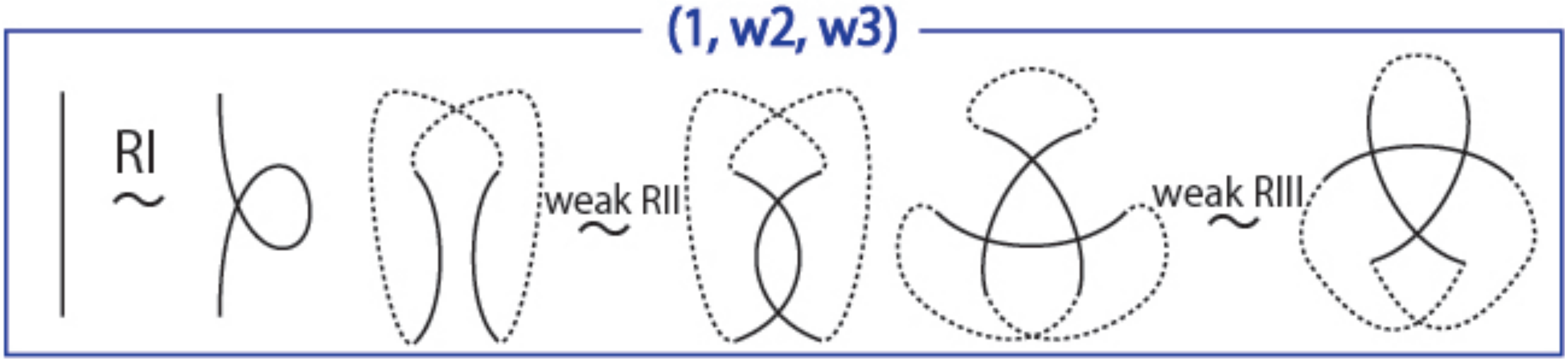
(1) $^{\forall}P_1, P_2$ は (1, s2, s3) で移り合う

(2) $^{\forall}P_1, P_2$ は (1, s2, w3) で移り合う

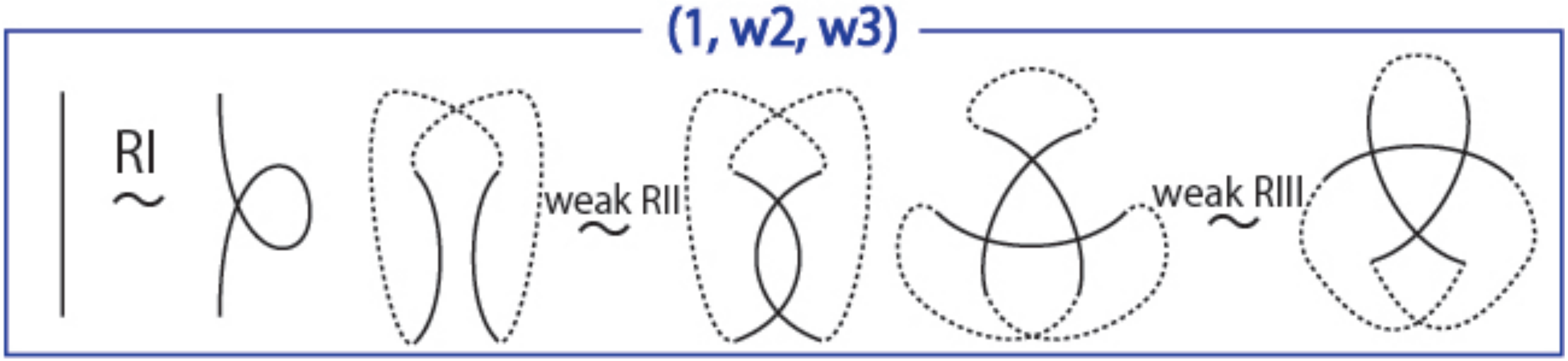
(3) $^{\forall}P_1, P_2$ は (1, w2, s3) で移り合う


(1, s2, s3)
RI
strong RII
strong RIII
(1, w2, w3)
RI
weak RII
weak RIII
(1, s2, w3)
RI
strong RII
weak RIII
(1, w2, s3)
RI
weak RII
strong RIII

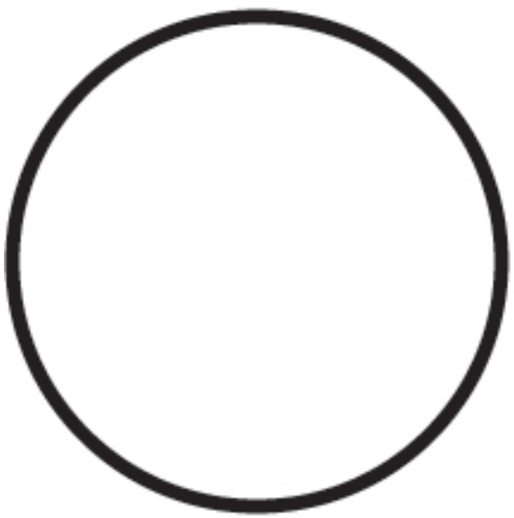

(1, w2, w3)
RI
weak RII
weak RIII


(1, w2, w3)
RI
weak RII
weak RIII

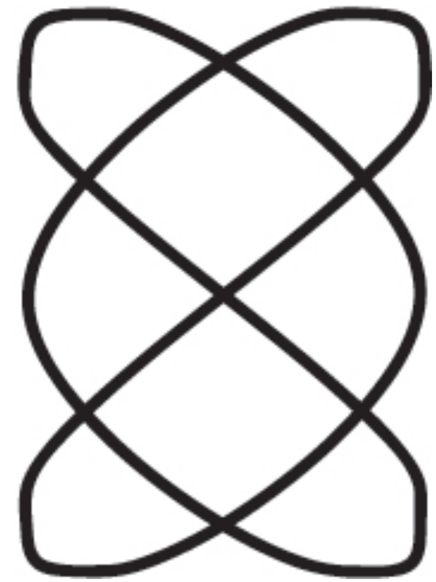
weak (1, 2, 3)

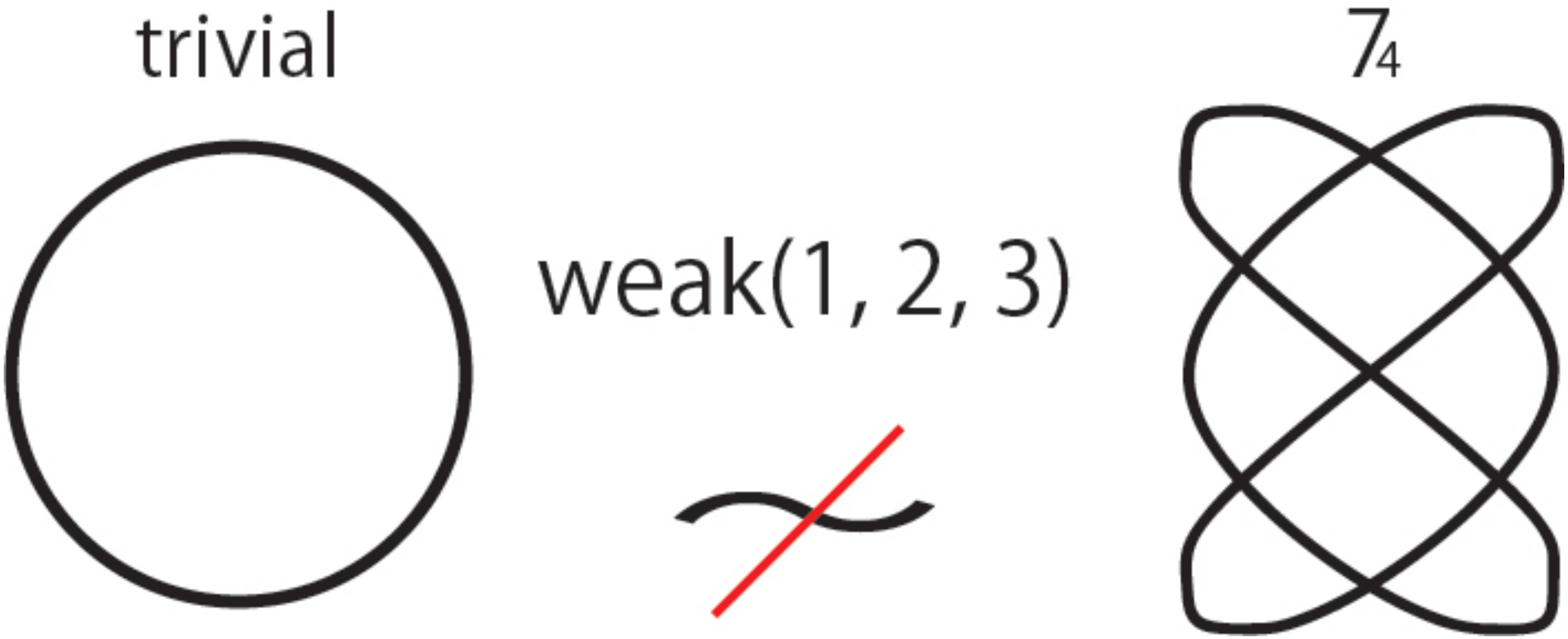


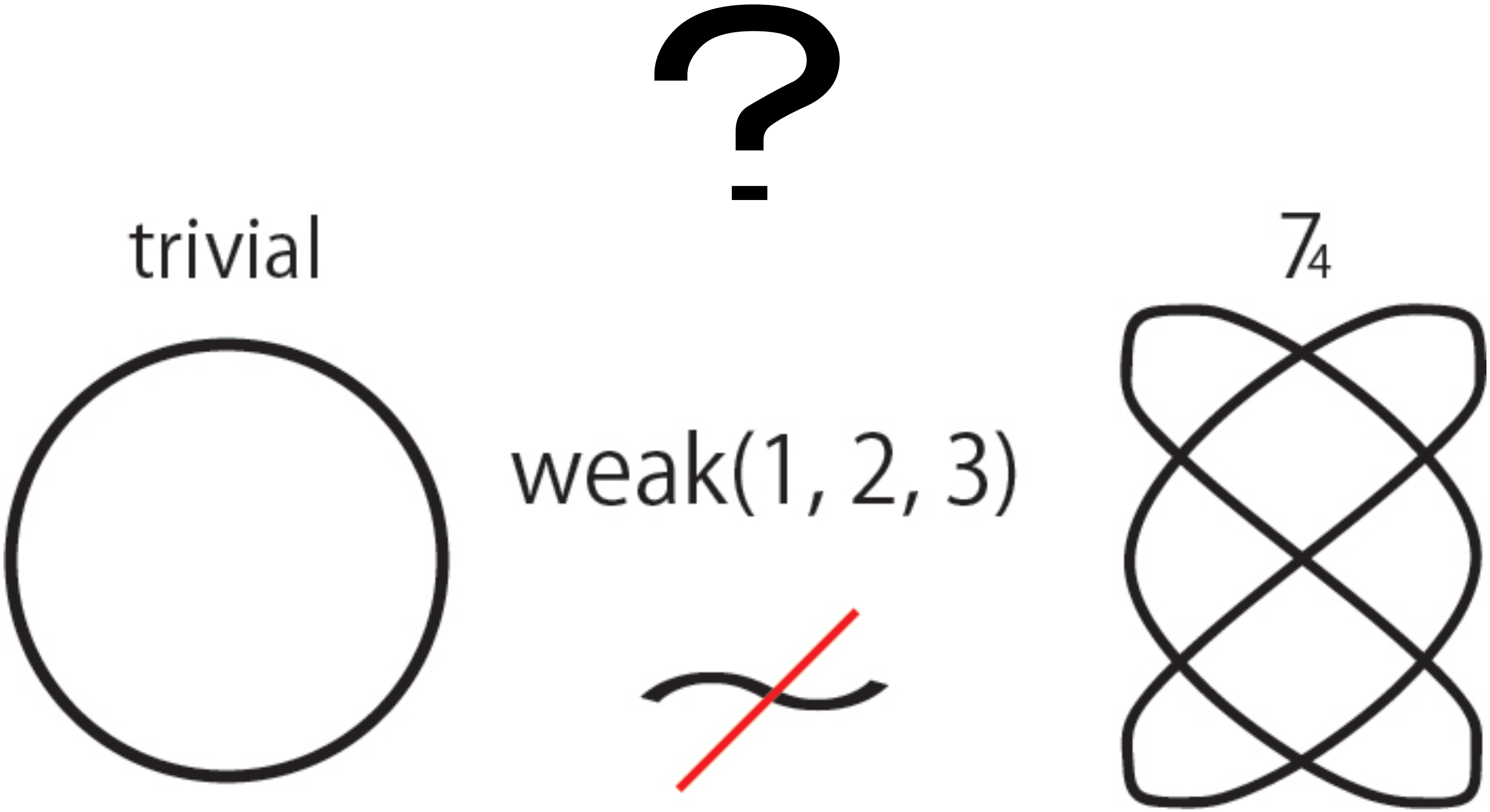
trivial



7_4

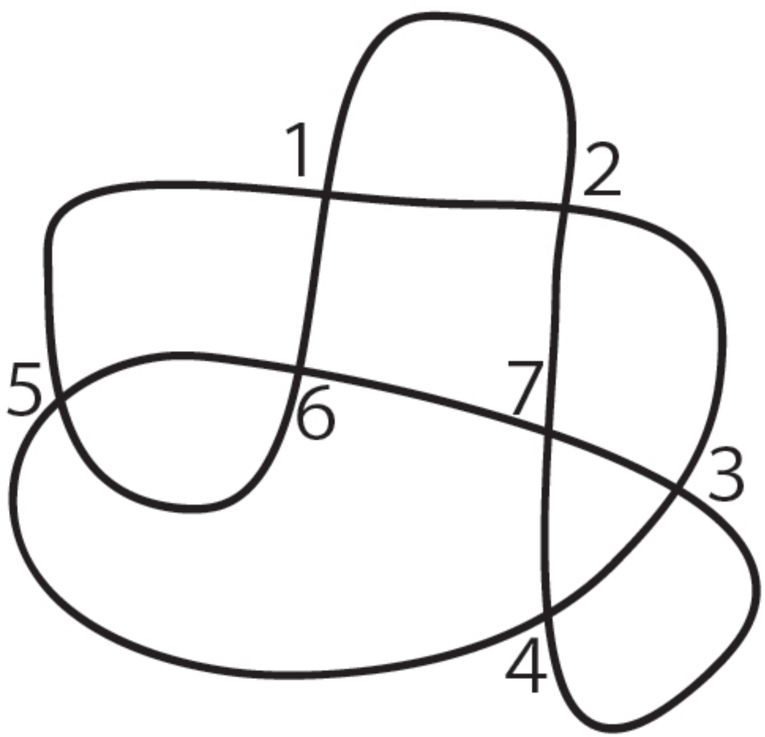



trivial
weak(1, 2, 3)
7_4

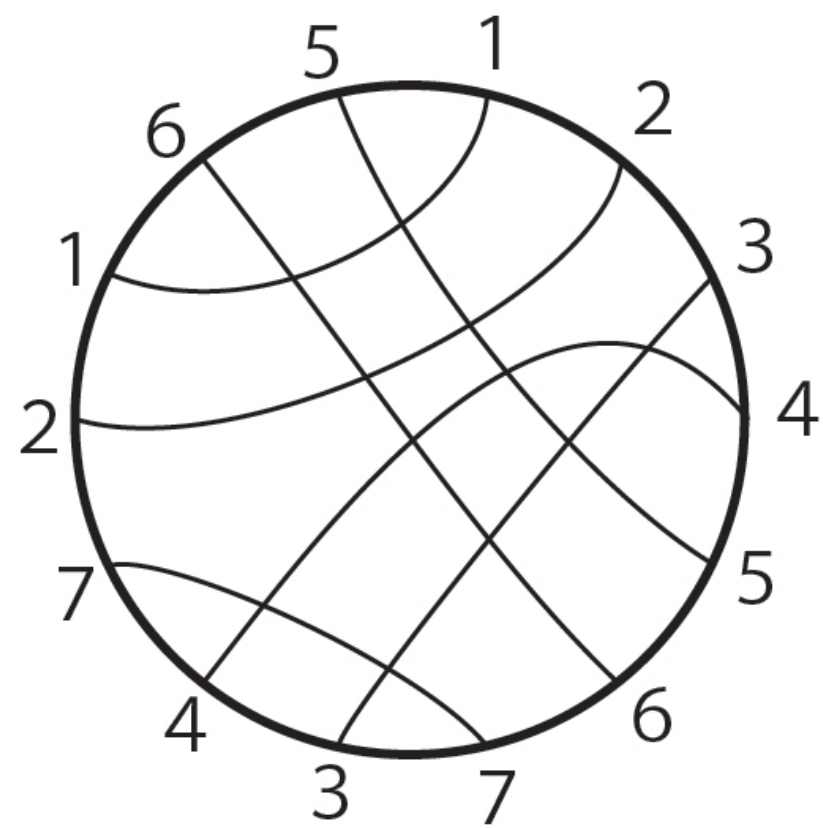

?
trivial
7_4
weak(1, 2, 3)
$\not\sim$

Definition

Knot projection の交点の逆像を
つなげたものをコード図という



Knot projection



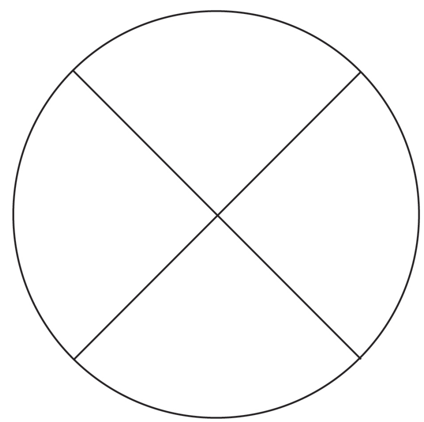
コード図

Hanaki (2009 OJM)

Theorem

(1) tr(P) = min{Pの コード図において、cross chords がなくなるまでコードを抜きとる数}

(2) tr(P) は偶数である



cross chords

Theorem

g(P) : P の canonical genus

W(P) : = tr(P) - 2g(P)　とする

Theorem

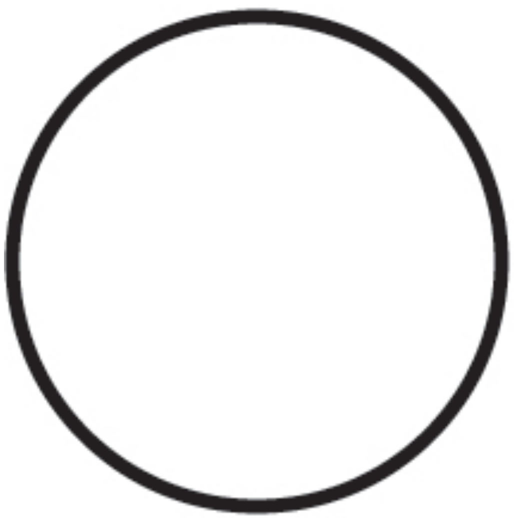
$g(P)$: P の canonical genus

$W(P) := tr(P) - 2g(P)$　とする

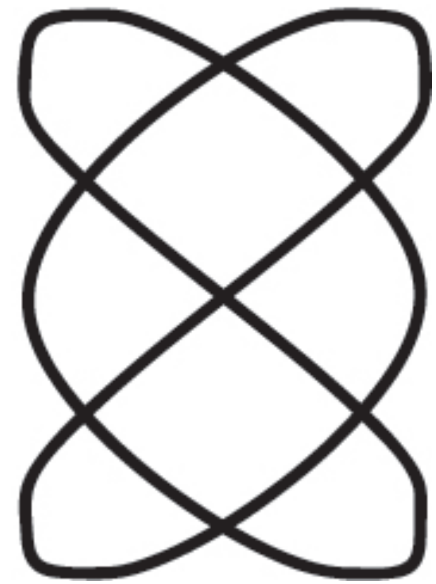
(1) $W(P)$ は weak(1, 2, 3) で不変である

$$W(P_1 \# P_2) = W(P_1) + W(P_2)$$

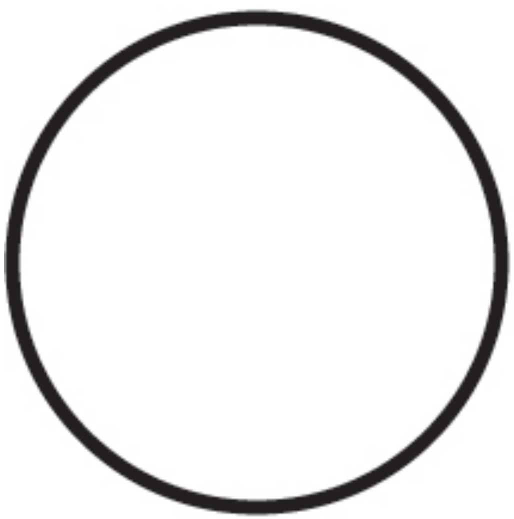
trivial



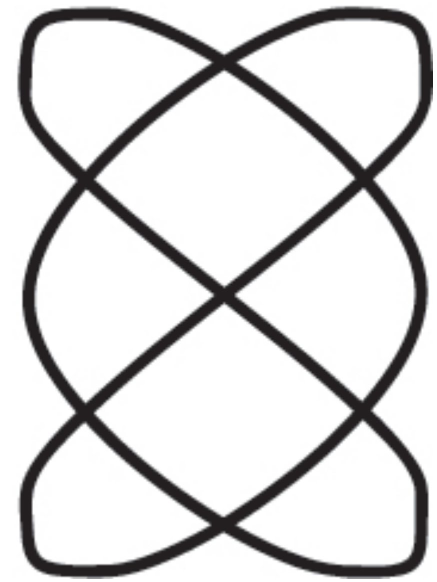
7_4



trivial



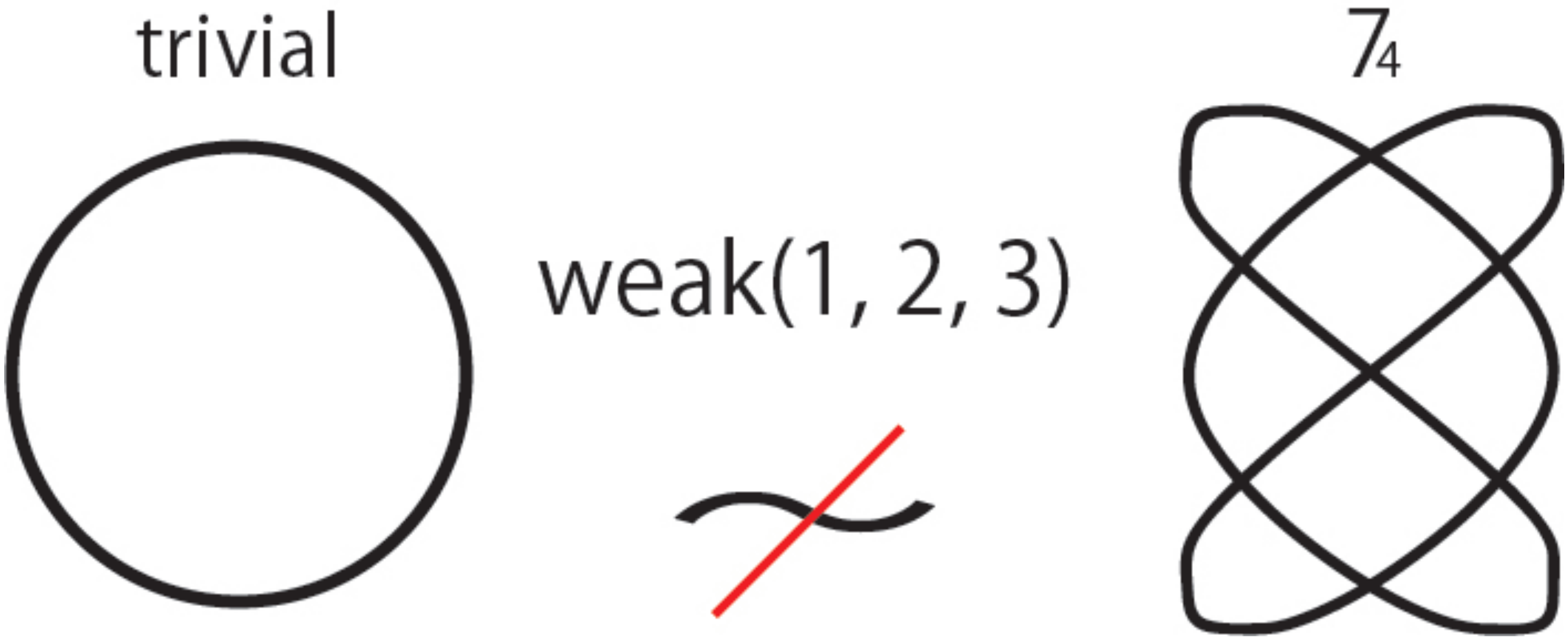
7_4



trivial

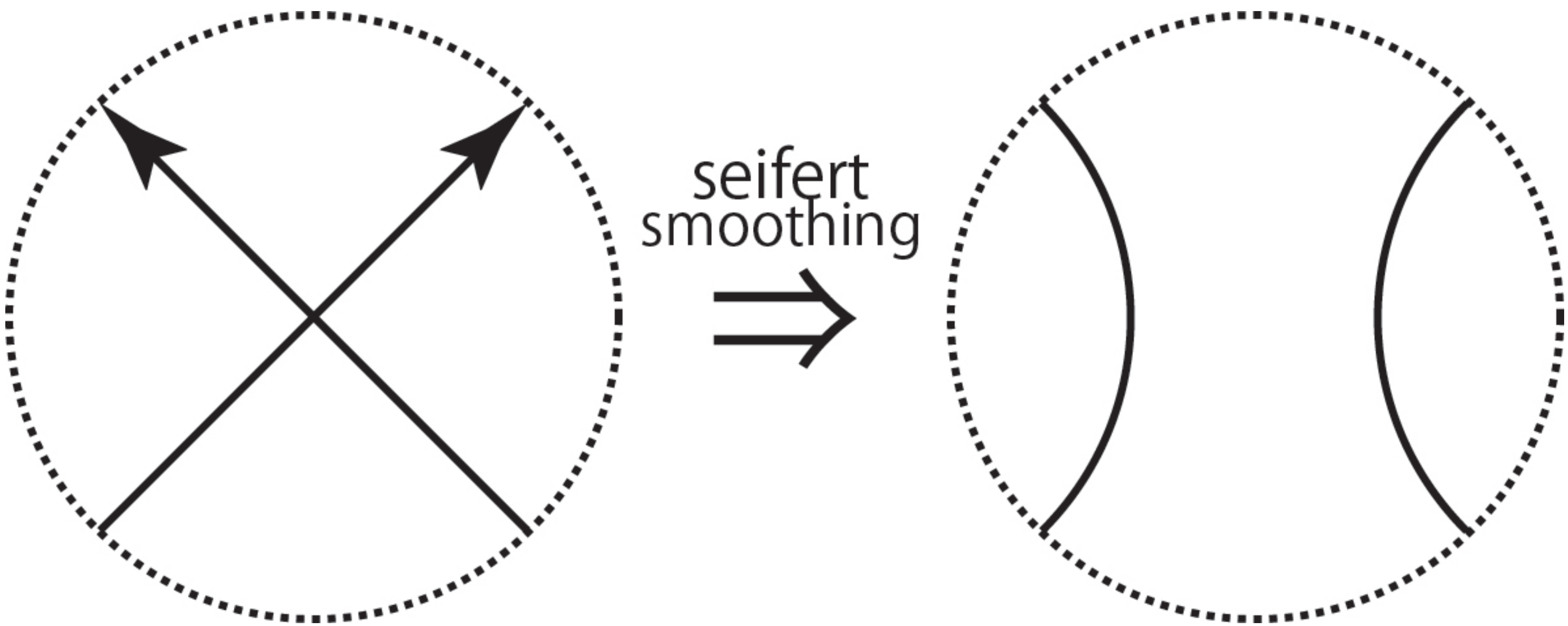
W(○) = 0

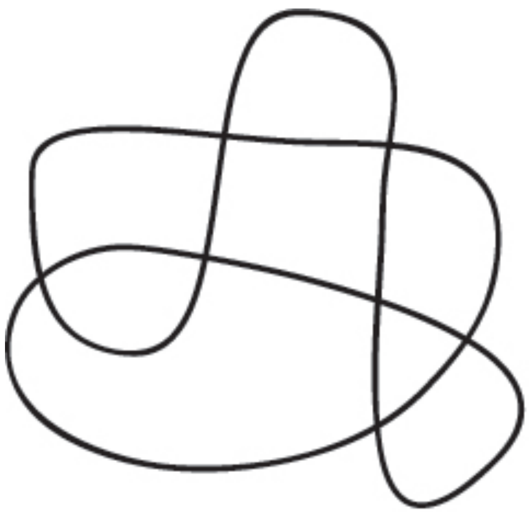
$W(7_4) = 2$


trivial
weak(1, 2, 3)
7
4

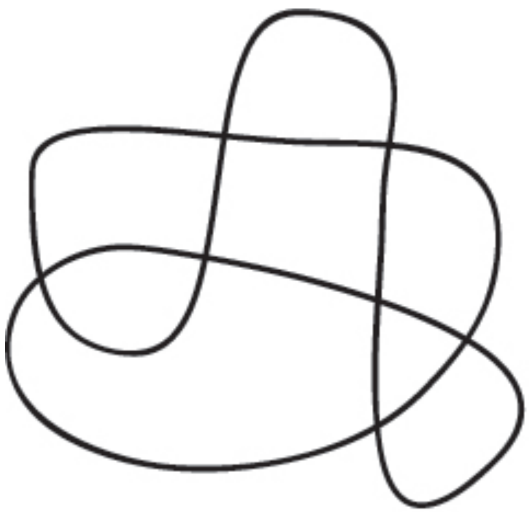
Definition

s(P) := P を seifert smoothing したときの circle数





P



P

Seifert smoothing

$\Rightarrow$


P
Seifert smoothing
s(P) = 4

Proof of Theorem

(1) $W(P) = tr(P) - 2g(P)$

Proof of Theorem

(1) $W(P) = tr(P) - 2g(P)$ (オイラー標数の計算)

Proof of Theorem

(1) $W(P) = tr(P) - 2g(P)$ (オイラー標数の計算)

$= tr(P) - (-s(P) + c(P) + 1)$

Proof of Theorem

(1) $W(P) = tr(P) - 2g(P)$ (オイラー標数の計算)

$= tr(P) - (-s(P) + c(P) + 1)$

$c(P) := P$ の交点数

Proof of Theorem

(1) $W(P) = tr(P) - 2g(P)$ (オイラー標数の計算)

$= tr(P) - (-s(P) + c(P) + 1)$

$c(P) := P$ の交点数

$= tr(P) + s(P) - c(P) - 1$

Proof of Theorem

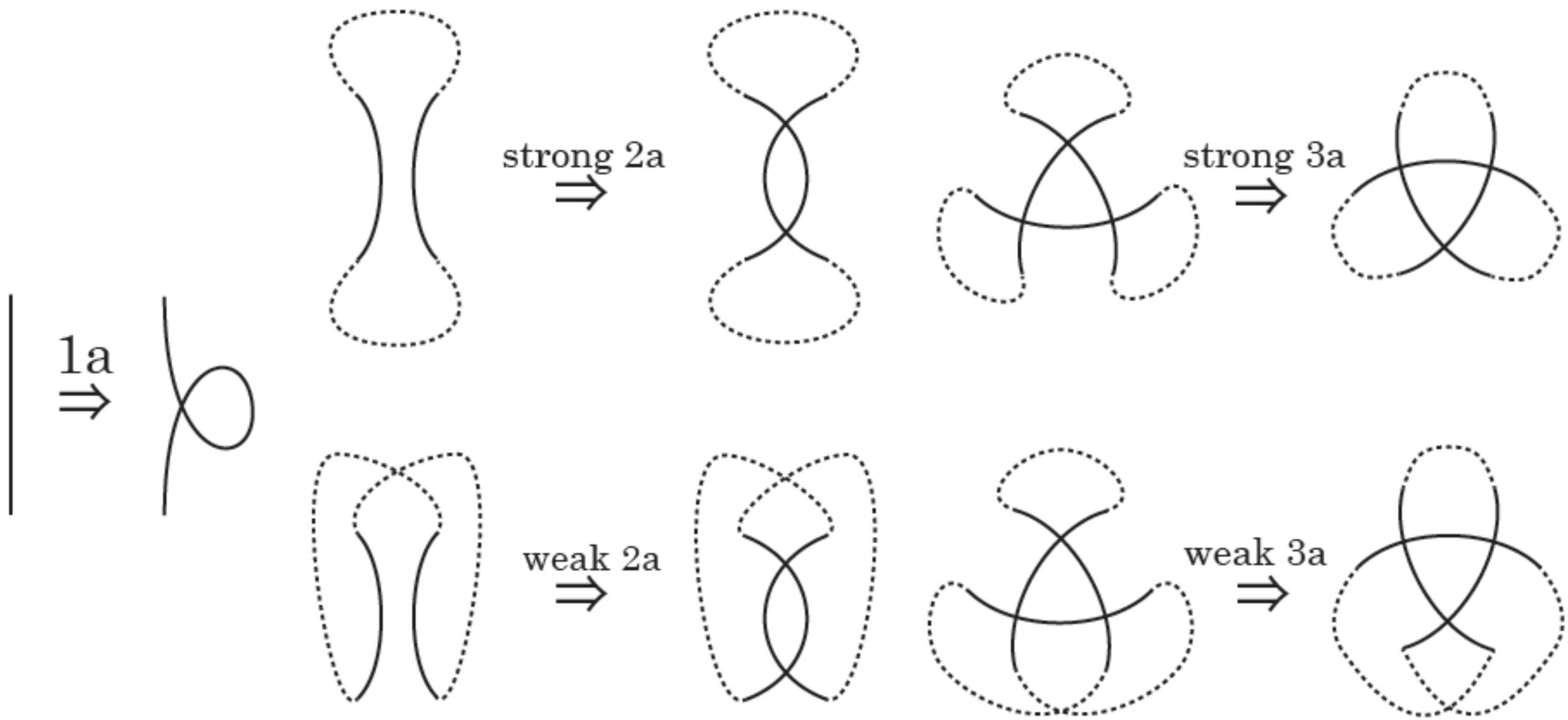
(1) W(P) = tr(P) - 2g(P) (オイラー標数の計算)

= tr(P) - (-s(P) + c(P) + 1)

c(P) : = P の交点数

= tr(P) + s(P) - c(P) - 1

tr(P) + s(P) - c(P) が weak(1, 2, 3) で不変であることを示せばよい


1a
strong 2a
strong 3a
weak 2a
weak 3a

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0

	1a	strong2a	weak2a	strong3a	weak3a
$\mathrm{tr}(P)$	0	0,2	2	0,2	0
$s(P)$	1	0,2	0	-2,0,2	0

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0
tr(P)+s(P)-c(P)	0	-2,0,2	0	-2,0,2,4	0

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0
tr(P)+s(P)-c(P)	0	-2,0,2	0	-2,0,2,4	0

tr(P) + s(P) - c(P) が weak(1, 2, 3) で不変

$$tr(P_1 \# P_2) = tr(P_1) + tr(P_2)$$

$$g(P_1 \# P_2) = g(P_1) + g(P_2)$$

$$W(P_1 \# P_2) =$$

$$W(P_1 \# P_2) = tr(P_1 \# P_2) - 2g(P_1 \# P_2)$$

$$W(P_1 \# P_2) = tr(P_1 \# P_2) - 2g(P_1 \# P_2)$$

$$= tr(P_1) + tr(P_2) - 2\{g(P_1) + g(P_2)\}$$

$$W(P_1 \# P_2) = tr(P_1 \# P_2) - 2g(P_1 \# P_2)$$

$$= tr(P_1) + tr(P_2) - 2\{g(P_1) + g(P_2)\}$$

$$= tr(P_1) - 2g(P_1) + tr(P_2) - 2g(P_2)$$

$$W(P_1 \# P_2) = tr(P_1 \# P_2) - 2g(P_1 \# P_2)$$

$$= tr(P_1) + tr(P_2) - 2\{g(P_1) + g(P_2)\}$$

$$= tr(P_1) - 2g(P_1) + tr(P_2) - 2g(P_2)$$

$$= W(P_1) + W(P_2)$$

Theorem

$g(P)$: P の canonical genus

$W(P) := tr(P) - 2g(P)$　とする

(1) $W(P)$ は weak(1, 2, 3) で不変である

$$W(P_1 \# P_2) = W(P_1) + W(P_2)$$

Theorem

(2) W(P) は 0以上の偶数である

Proof of Theorem

(2) W(P) は 0以上の偶数である

Proof of Theorem

(2) W(P) は 0以上の偶数である

$$W(P) = tr(P) - 2g(P)$$

Proof of Theorem

(2) W(P) は 0以上の偶数である

$$W(P) = tr(P) - 2g(P)$$

偶　　偶

Proof of Theorem

(2) W(P) は 0以上の偶数である

$$W(P) = tr(P) - 2g(P)$$

偶　　偶

(Henrich, Macnaughton, Narayan, Pechenik, and Townsend, JKTR, 2011)

Proof of Theorem

(2) W(P) は 0以上の偶数である

$$W(P) = tr(P) - 2g(P)$$

偶　偶

(Henrich, Macnaughton, Narayan, Pechenik, and Townsend, JKTR, 2011)

より $tr(P) \geqq 2g(P)$

Theorem

(3)

(i) $c(P) = 0$ のとき $W(P) = 0$

(ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

(iii) $W(P) = c(P) - 1 \Leftrightarrow c(P) = 1$

Proof of Theorem

(3)

(i) $c(P) = 0$ のとき $W(P) = 0$

Proof of Theorem

(3)

(i) $c(P) = 0$ のとき $W(P) = 0$

$$W(P) = tr(P) - 2g(P)$$

$$= 0$$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

$s(P) \leq c(P) + 1$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$tr(P) + s(P) \leq 2c(P)$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

$s(P) \leq c(P) + 1$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

$s(P) \leq c(P) + 1$

$tr(P) + s(P) \leq 2c(P)$

$tr(P) + s(P) - c(P) \leq c(P)$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

$s(P) \leq c(P) + 1$

$tr(P) + s(P) \leq 2c(P)$

$tr(P) + s(P) - c(P) \leq c(P)$

$tr(P) + s(P) - c(P) - 1 \leq c(P) - 1$

Proof of Theorem

(3) (ii) $1 \leq c(P)$ のとき $0 \leq W(P) \leq c(P) - 1$

$1 \leq c(P)$ のとき

$tr(P) \leq c(P) - 1$
(Hanaki, OJM, 2010)

$s(P) \leq c(P) + 1$

$tr(P) + s(P) \leq 2c(P)$

$tr(P) + s(P) - c(P) \leq c(P)$

$tr(P) + s(P) - c(P) - 1 \leq c(P) - 1$

$W(P) \leq c(P) - 1$

Proof of Theorem

(3) (iii) $W(P) = c(P) - 1 \Leftrightarrow c(P) = 1$

Proof of Theorem

(3) (iii) $W(P) = c(P) - 1 \Leftrightarrow c(P) = 1$

$tr(P) = c(P) - 1$

Proof of Theorem

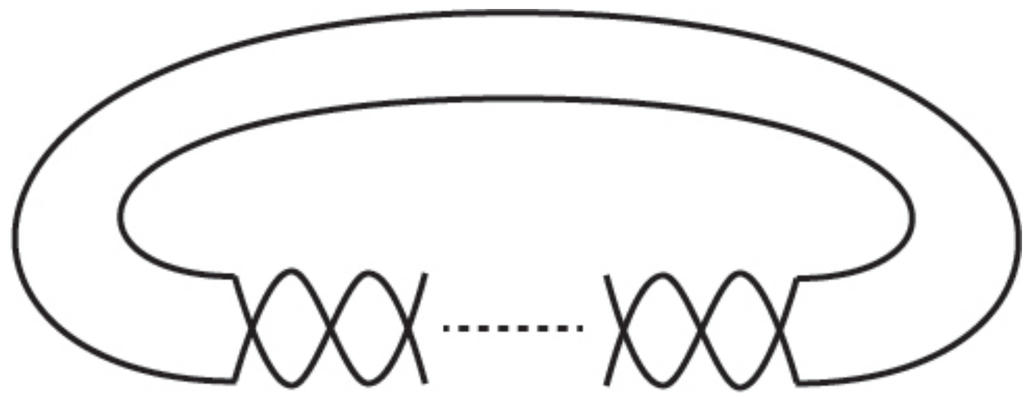
(3) (iii) $W(P) = c(P) - 1 \Leftrightarrow c(P) = 1$

$tr(P) = c(P) - 1$
(Hanaki, OJM, 2010)

Proof of Theorem

(3) (iii) W(P) = c(P) - 1 $\Leftrightarrow$ c(P) = 1

tr(P) = c(P) - 1
(Hanaki, OJM, 2010)

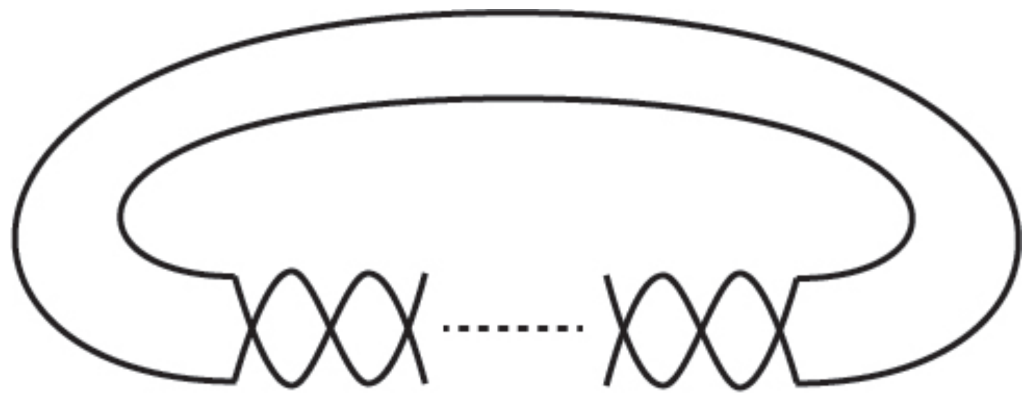


(2, P) torsu knot

Proof of Theorem

(3) (iii) W(P) = c(P) - 1 ⇔ c(P) = 1

tr(P) = c(P) - 1
(Hanaki, OJM, 2010)



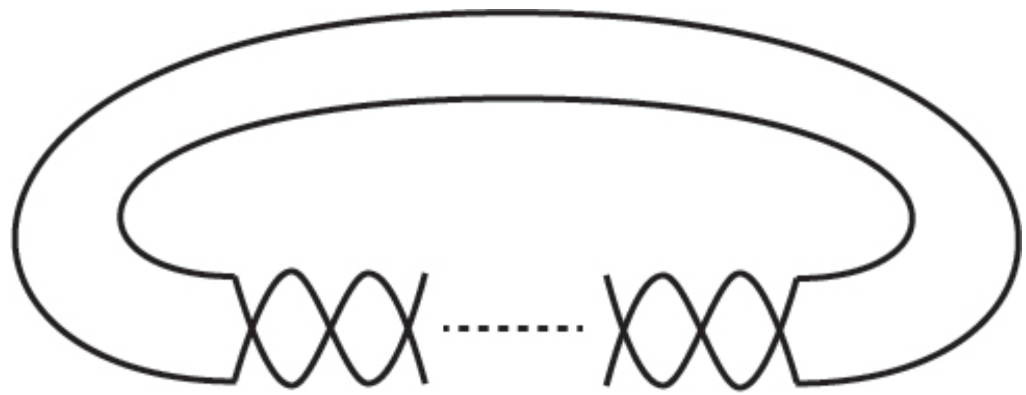
(2, P) torsu knot

s(P) = c(P) + 1

Proof of Theorem

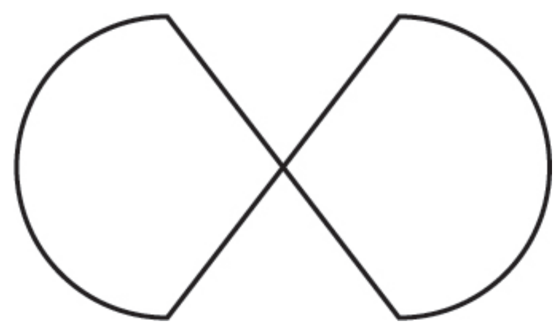
(3) (iii) $W(P) = c(P) - 1 \Leftrightarrow c(P) = 1$

$tr(P) = c(P) - 1$
(Hanaki, OJM, 2010)



(2, P) torsu knot

$s(P) = c(P) + 1$



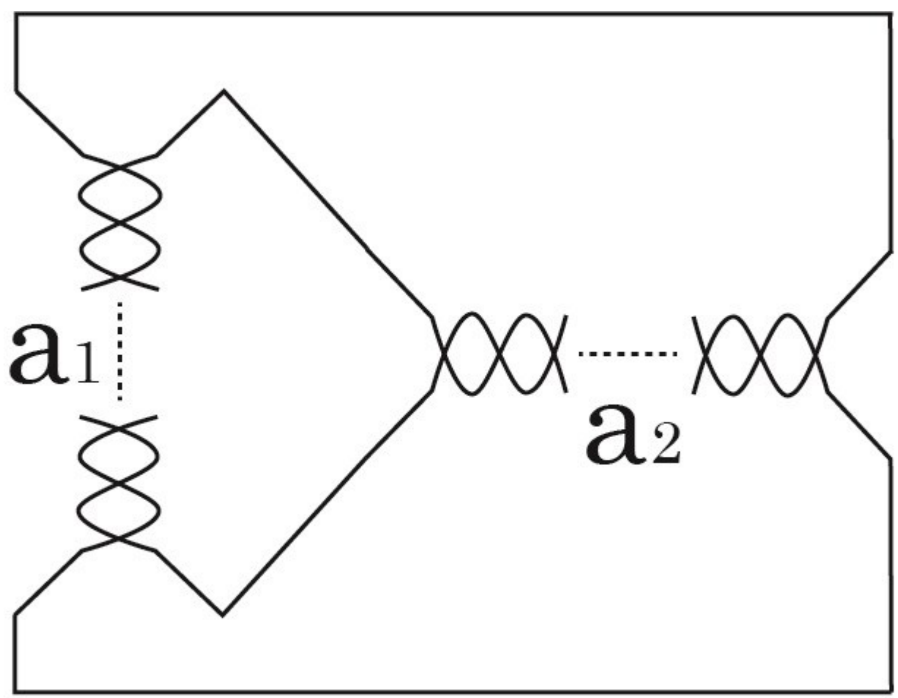
Theorem

(4) 任意の 0 以上の偶数 m に対し
W(P) = m となる prime な P が存在する

a_1, a_2 : 偶数　$a_1 \geq a_2 \geq 2$

$c(P) = a_1 + a_2$, $s(P) = a_1 + a_2 - 1$, $tr(P) = a_2$

$W(P_1) = a_2 - 2$

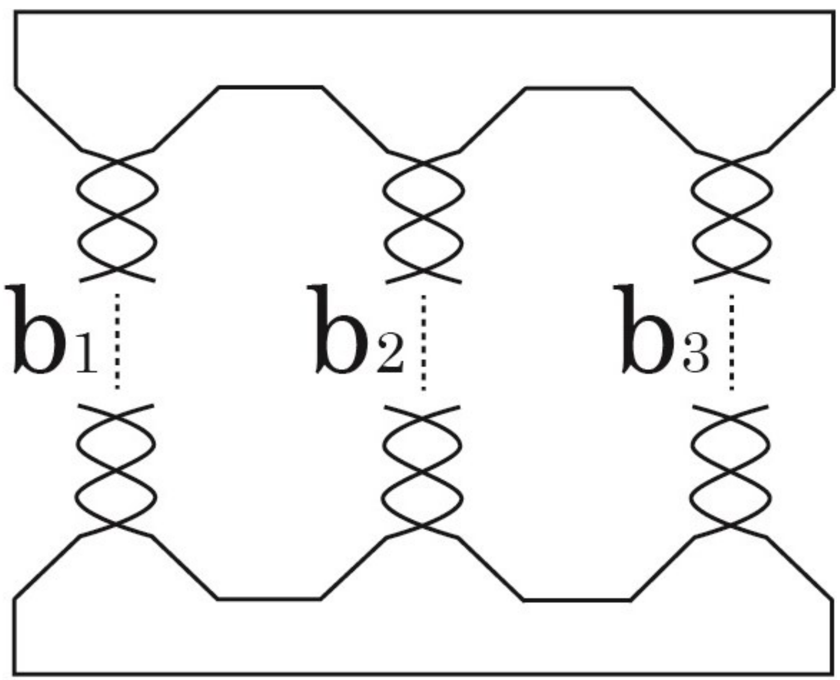


P_1

b_1, b_2, b_3：奇数　$b_1 \geq b_2 \geq b_3 \geq 1$

$c(P) = b_1 + b_2 + b_3$, $s(P) = b_1 + b_2 + b_3 - 1$
$tr(P) = b_2 + b_3$

$W(P_2) = b_2 + b_3 - 2$



P_2

Theorem

(4) 任意の 0 以上の偶数 m に対し
W(P) = m となる prime な P が存在する

Question 1

weak(1, 2, 3)

$W(P) = 0$ かつ $P \not\sim$ ○

となる P はあるか？

Question 2

(1) 1回の strong 2a で W(P) が -2 変化する例はあるか？

(2) 1回の strong 3a で W(P) が +4 変化する例はあるか？

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0
tr(P)+s(P)-c(P)	0	-2,0,2	0	-2,0,2,4	0

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0
tr(P)+s(P)-c(P)	0	-2,0,2	0	-2,0,2,4	0

	1a	strong2a	weak2a	strong3a	weak3a
tr(P)	0	0,2	2	0,2	0
s(P)	1	0,2	0	-2,0,2	0
c(P)	1	2	2	0	0
tr(P)+s(P)-c(P)	0	-2,0,2	0	-2,0,2,4	0

? ?

Thank you for listening