Knot projections with reductivity two

(Topology Appl. **193** (2015) 290-301)

伊藤昇（早稲田大学高等研究所）

瀧村祐介氏(学習院中等科)
との共同研究

結び目の数学VIII

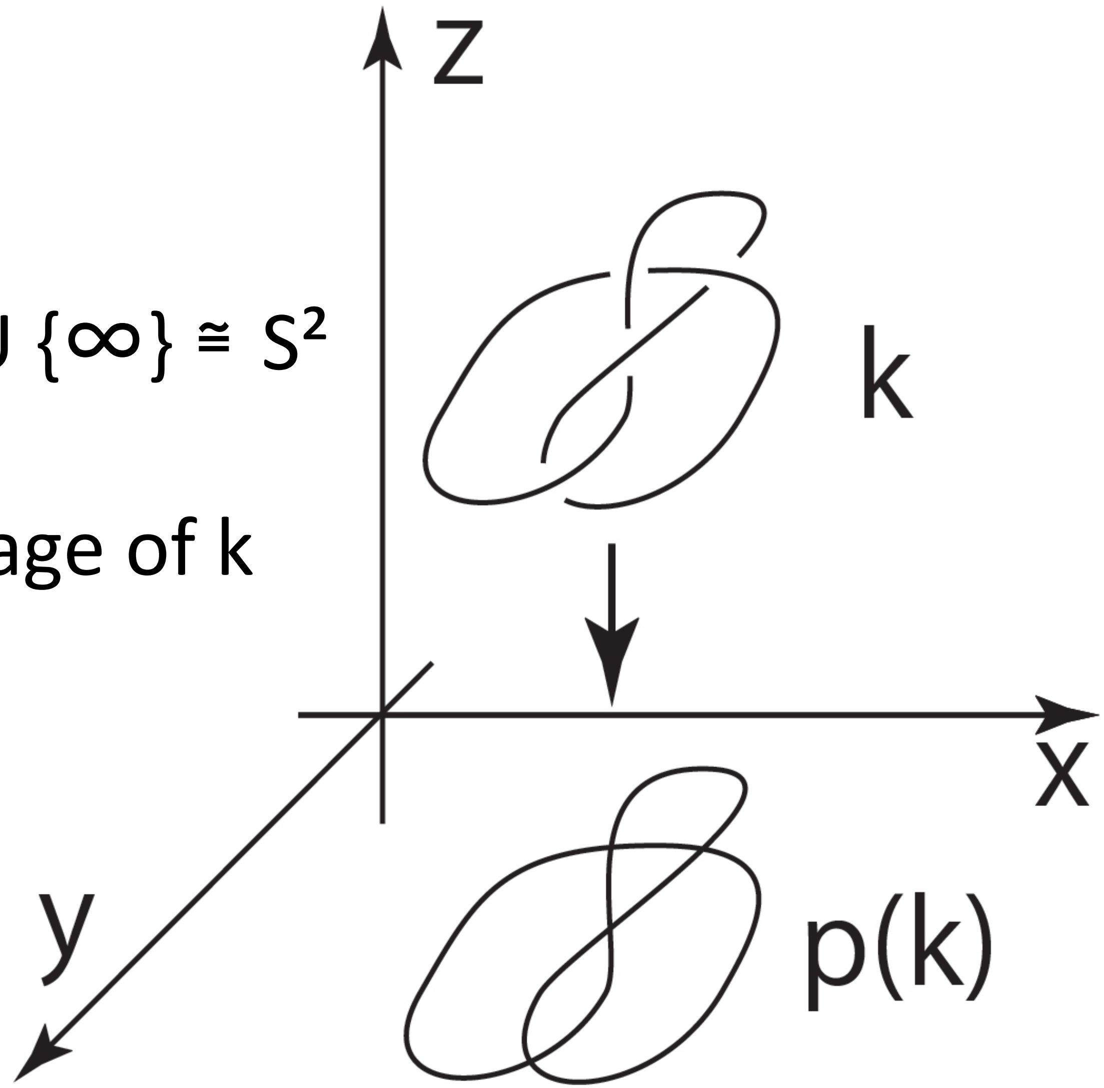
Definition

k: knot in R^3

$p : R^3 \longrightarrow R^2 \subset R^2 \cup \{\infty\} \cong S^2$

p(k) : projection image of k

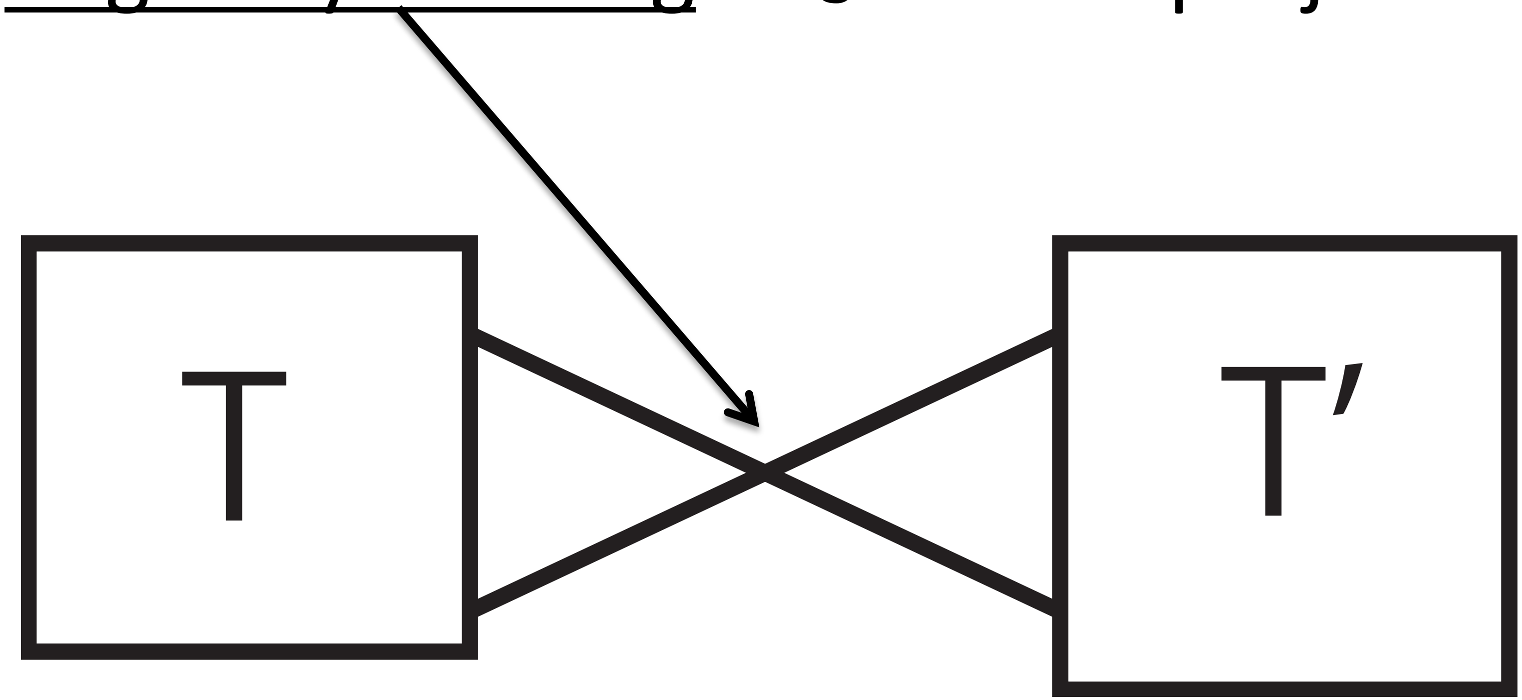
Denote p(k) by P.



Definition

Reduced knot projection (= irreducible)

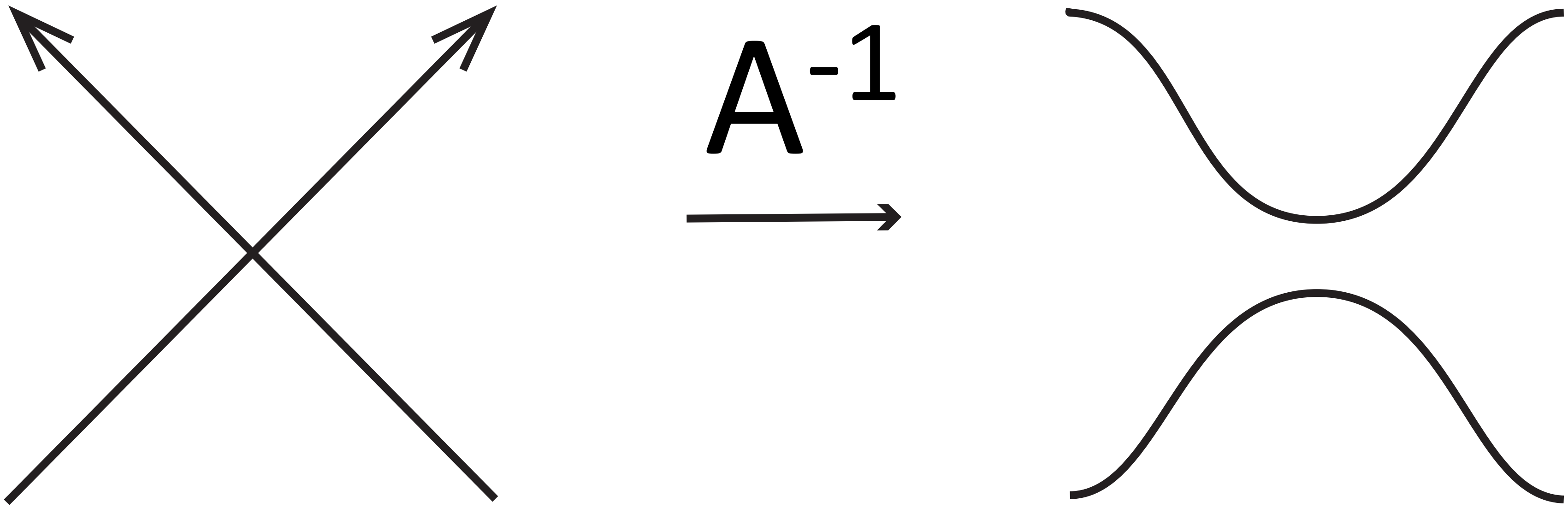
= nugatory crossingがないknot projection



Definition 既約度(Reductivity)[清水]

P: knot projection.

r(P) = min{♯A^{-1} to obtain reducible P’}



Motivation [清水の結果]

オイラー数の計算からreduced knot projectionは２辺形か３辺形をもつ

（i.e.,２辺形と３辺形は不可避集合をなす）

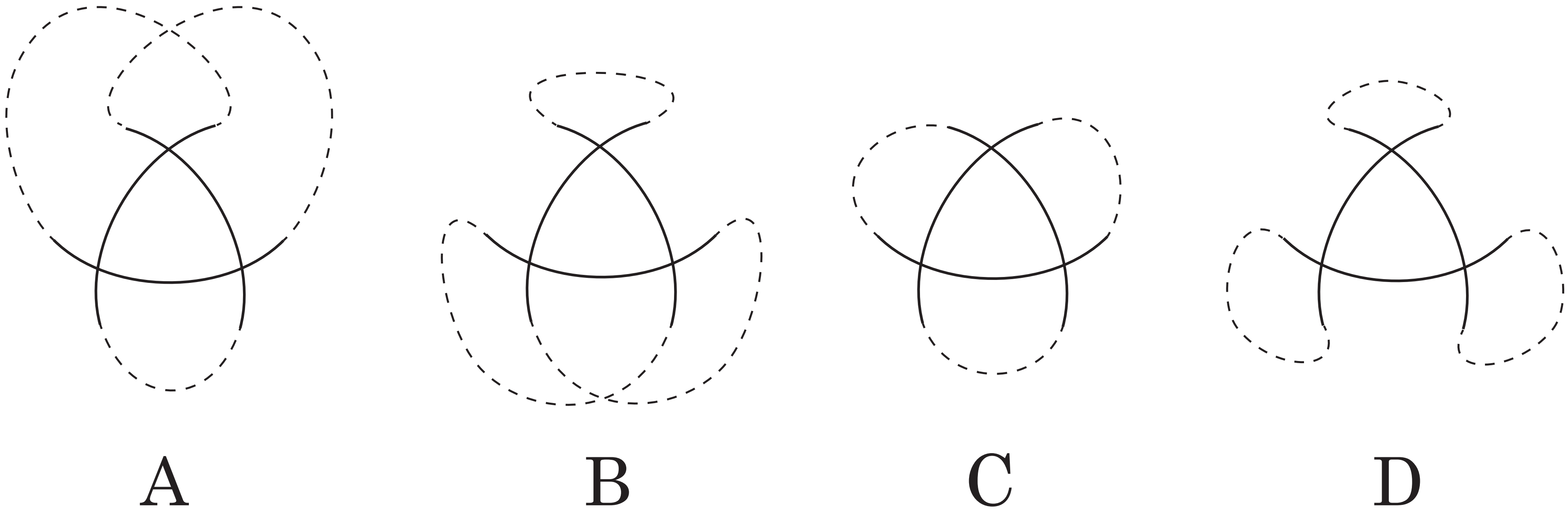
⇒ 定理[清水]　$r(P) \leqq 4 \quad (\forall P)$

Motivation

[Shimizu, Topology Appl. **196** (2015)]

仮に２辺形とD型３辺形以外が不可避集合をなすならば、

$r(P) \leqq 3 \ (\forall P)$　　である。

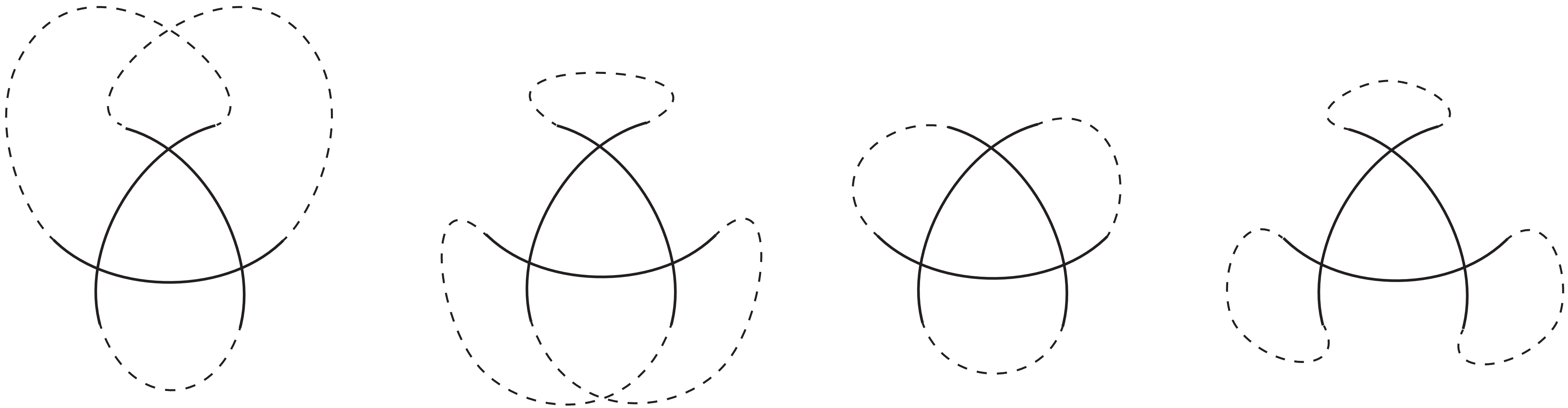


Motivation

[Shimizu, Topology Appl. **196** (2015)]

２辺形とD型３辺形以外が不可避集合をなす<不可避集合の予想>

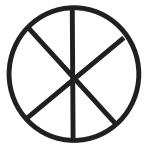
$\Rightarrow \quad r(P) \leqq 3 \ (\forall P)$ <既約度の予想>

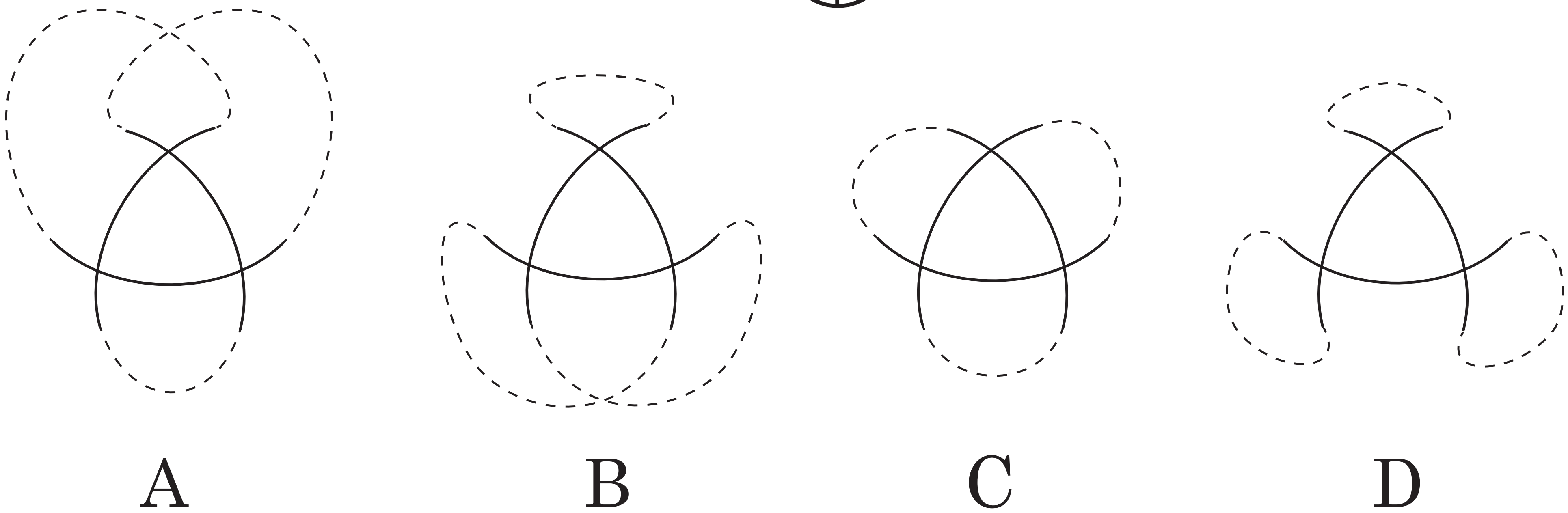


A　　B　　C　　D

Motivation [観察]

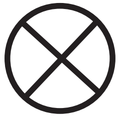
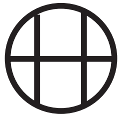
仮に２辺形とD型３辺形以外が不可避集合をなすならば、

1辺形も2辺形も持たないprime-reducedなknot projectionは  をもつ

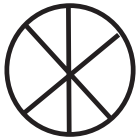


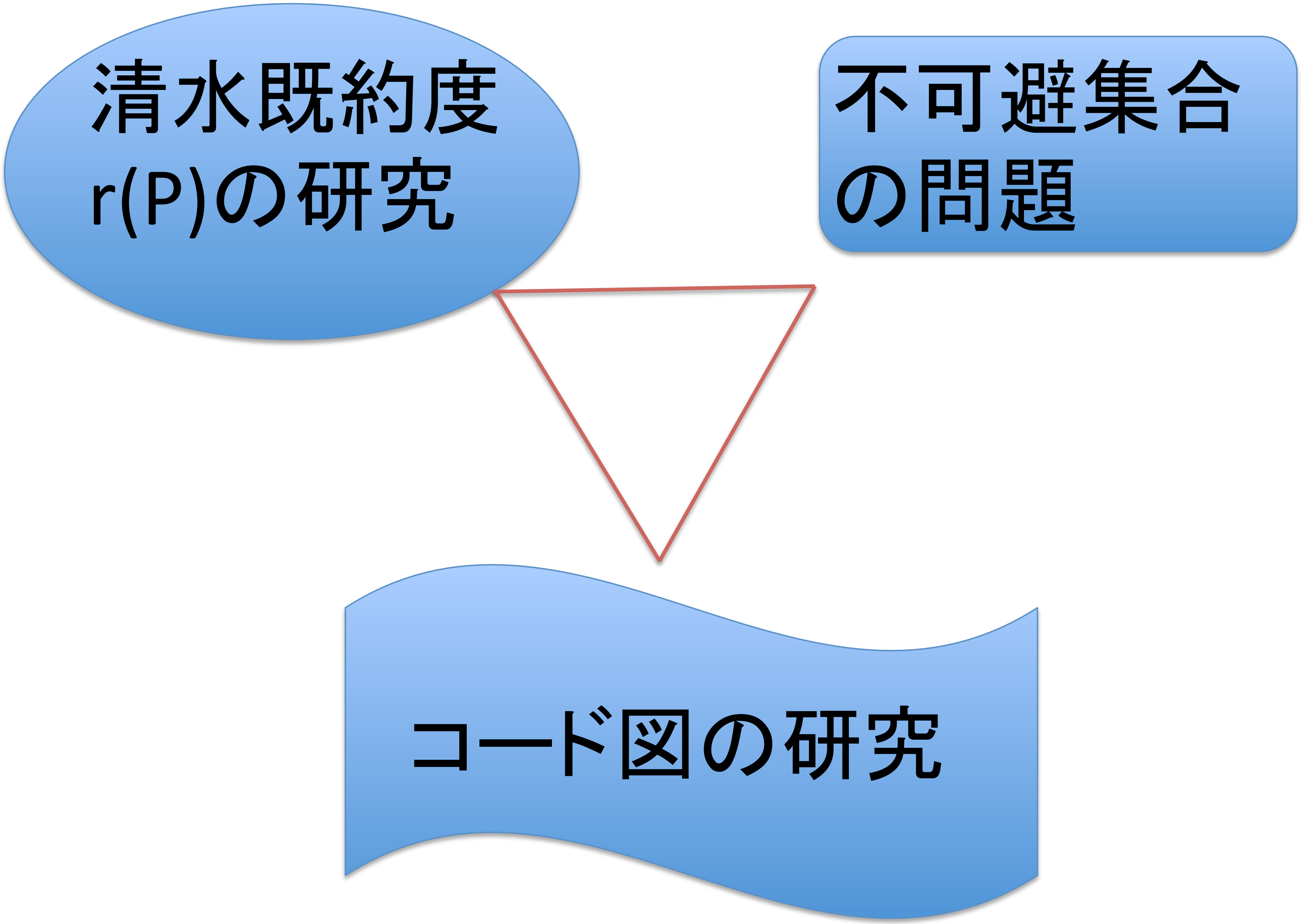
もう一つのMotivation

[坂本-谷山2013], [谷山1989]では

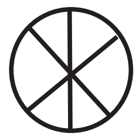
 ,  をもつ、もたないの

必要十分条件が与えられている。

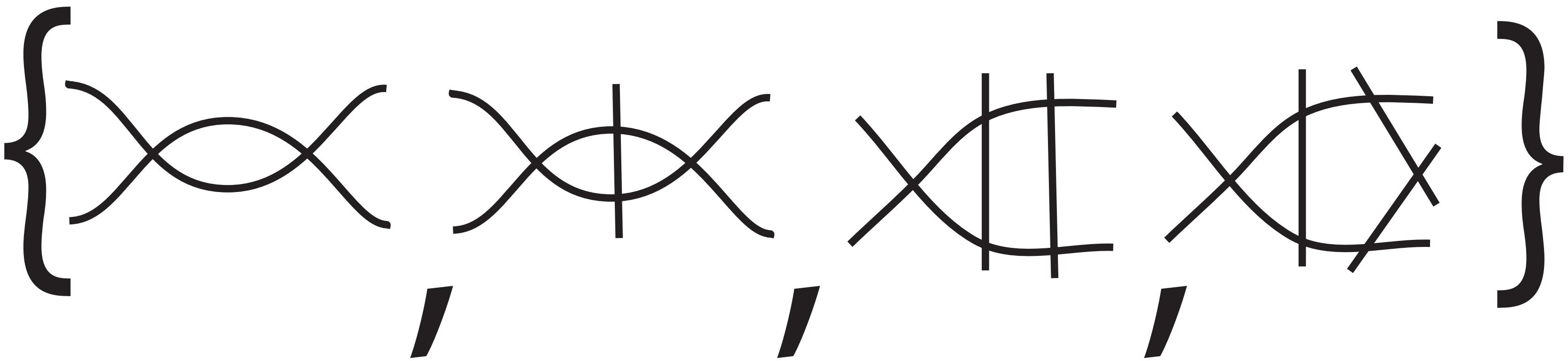
 ではどうか？


清水既約度
r(P)の研究
不可避集合
の問題
コード図の研究

Motivation

定理[I.-Takimura, J. MSJ, in Press]
１辺形も2辺形も持たないprime reducedなknot projectionは、コード図に  をもつ.

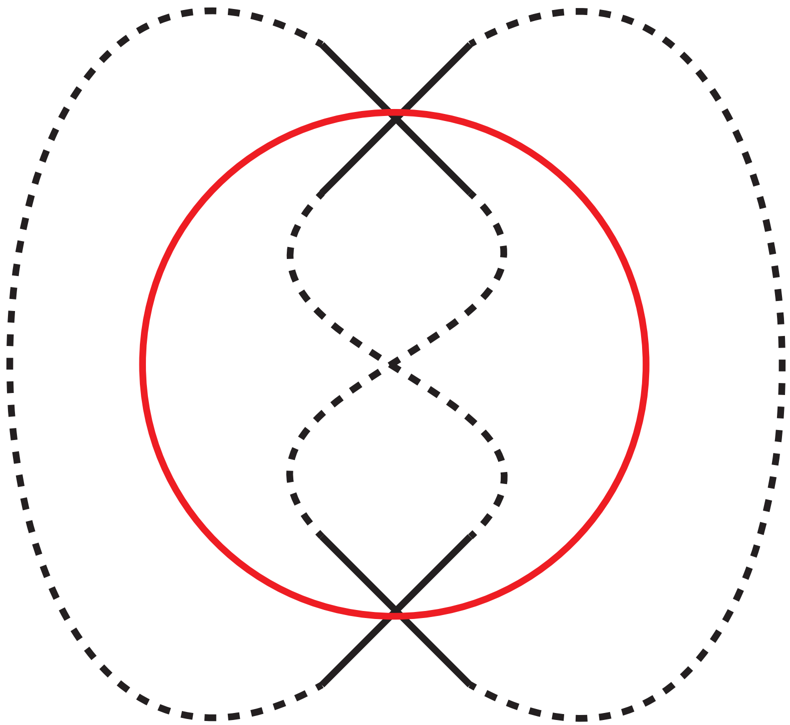
証明は別の清水の不可避集合を用いた



今回の結果(1)[I.-Takimura]

knot projectionが清水既約度１をもつ

⇔図のような、ちょうど２交点と交わる simple circleがとれる

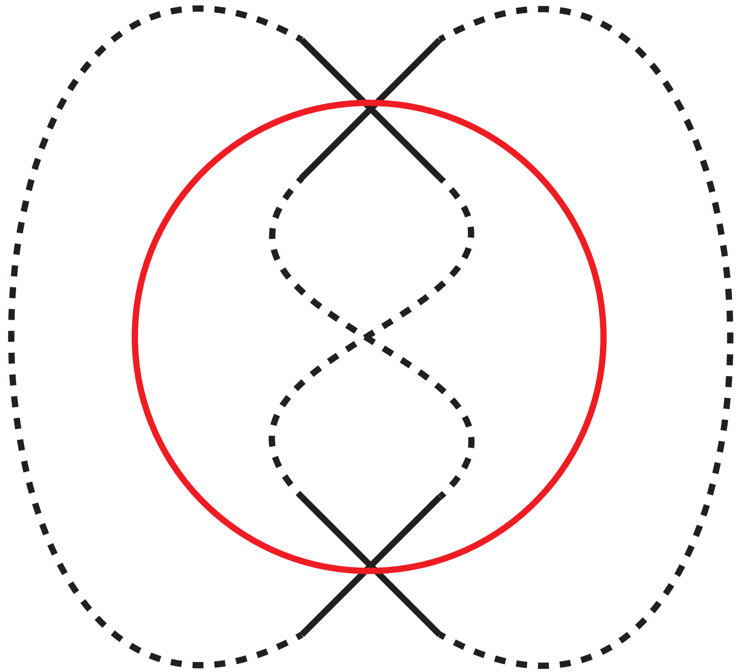


今回の結果(1) [I.-Takimura]

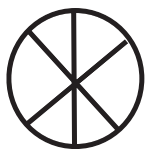
reduced knot projectionがr(P)=1

⇔図のような、ちょうど２交点と交わる

simple circleがとれる.

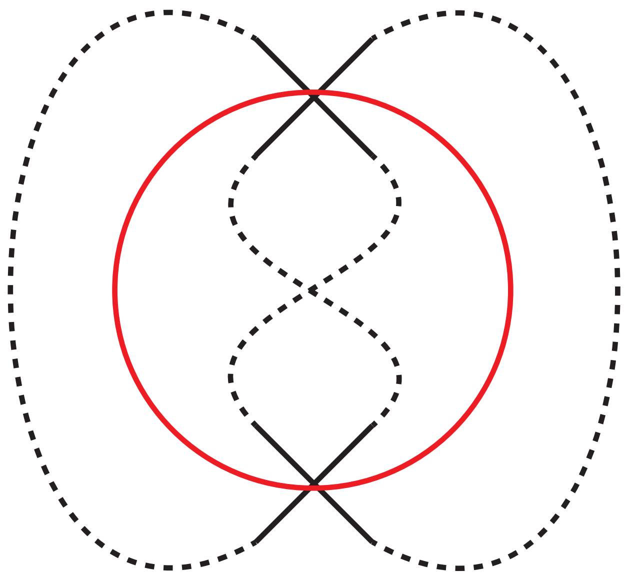


Cor.

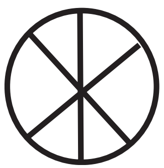
r(P)=1⇒Pは  をもつ

今回の結果(1) [I.-Takimura]

reduced knot projectionがr(P)=1
⇔図のような、ちょうど2交点と交わる
simple circleがとれる.

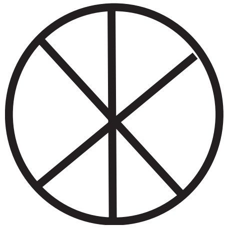


Cor.

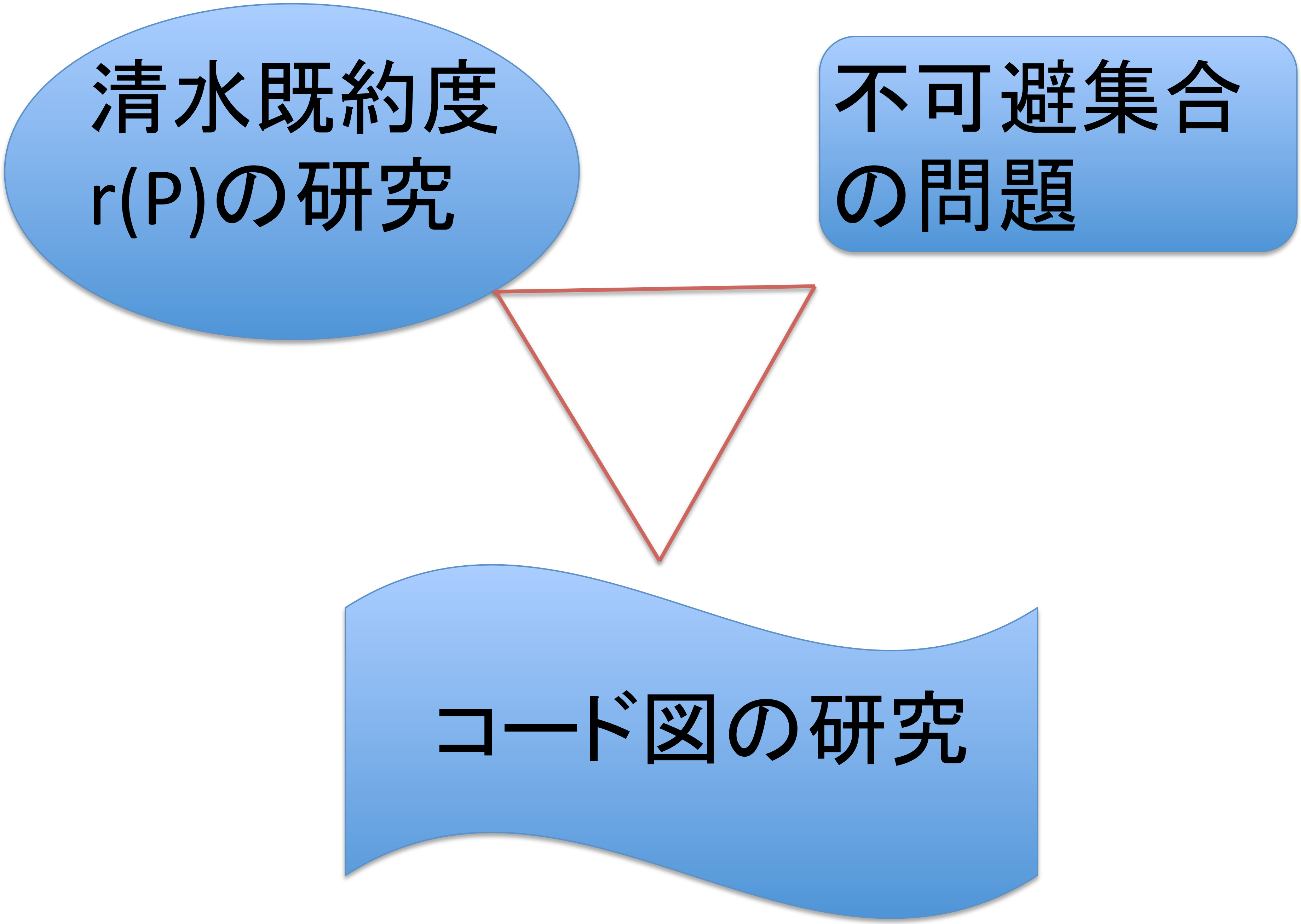
 を持たない⇒1<r(P).

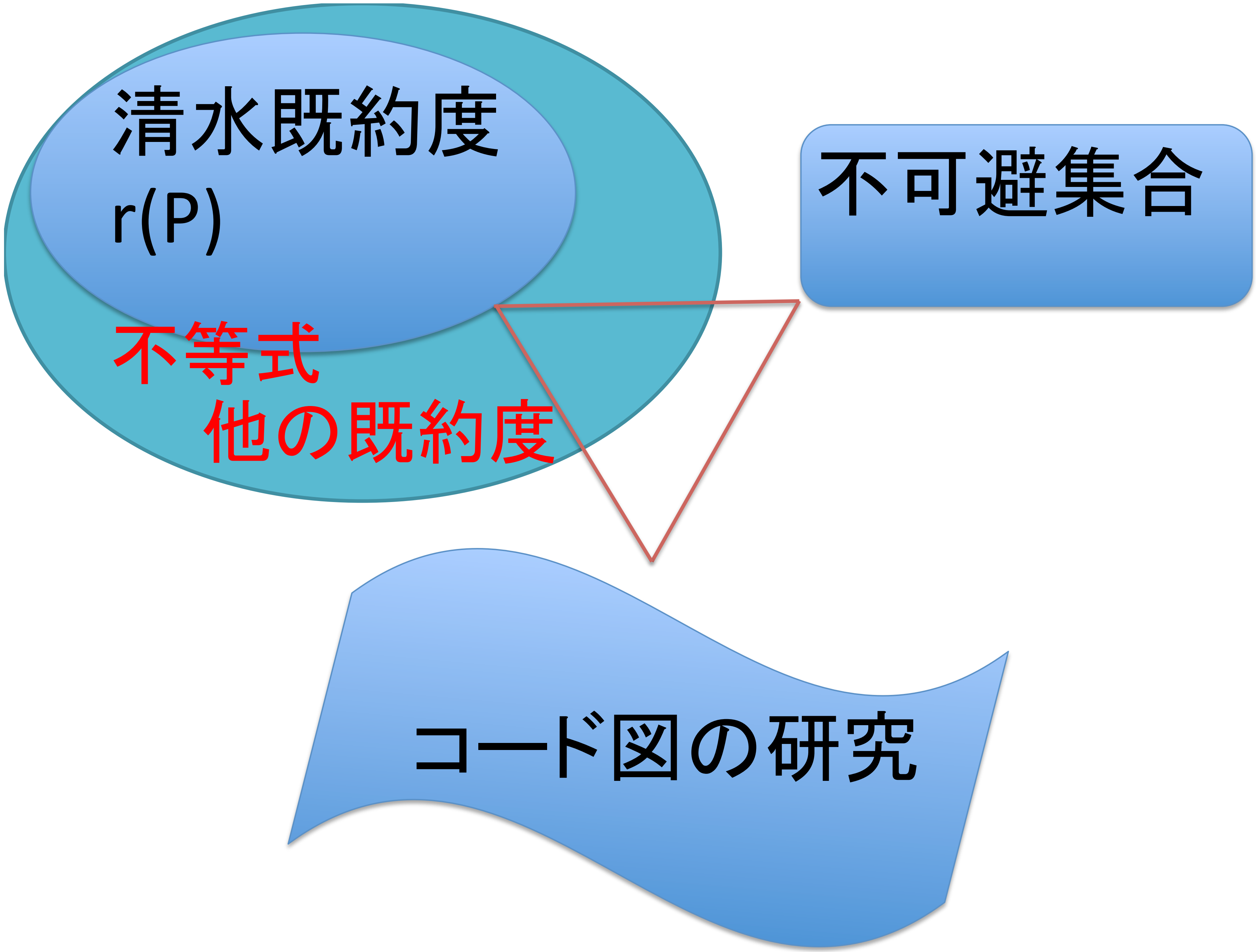
(ごく最近の別の結果から実はr(P)=2.)

(ちょっと余談) ごく最近の結果

knot projection Pのコード図
が、をもたない
⇒

Pは1辺形か２辺形をもつ。


清水既約度
r(P)の研究
不可避集合
の問題
コード図の研究

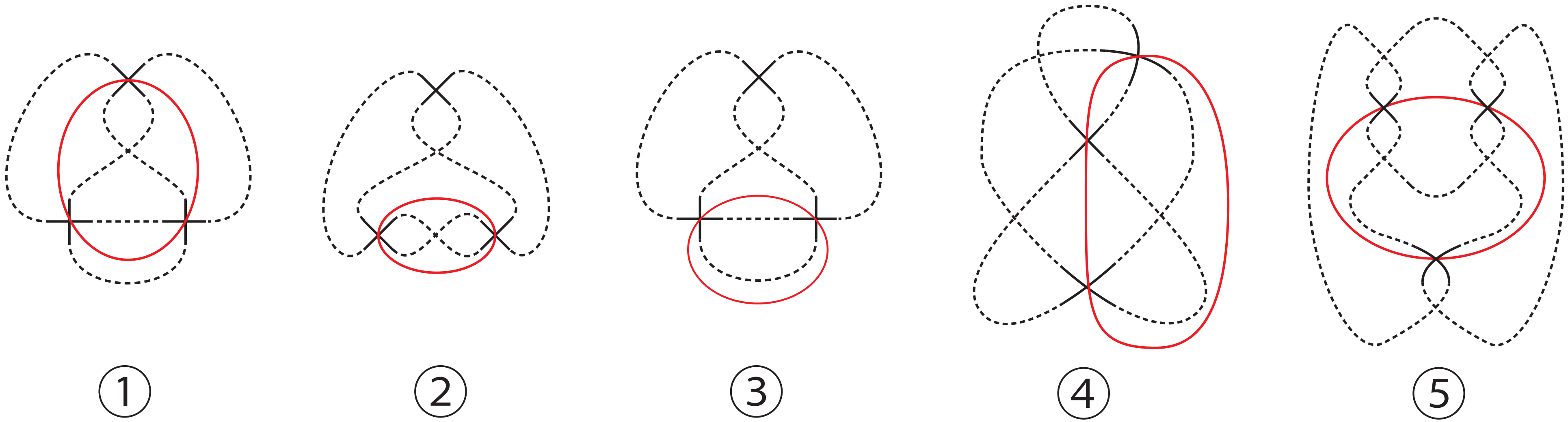

清水既約度
r(P)
不等式
他の既約度
不可避集合
コード図の研究

結果(2)[I.-Takimura, Topology Appl.2015]

P: reduced

r(P)=2

⇔Pと2or3交点で交わる下記の円周が存在かつr(P)≠1.

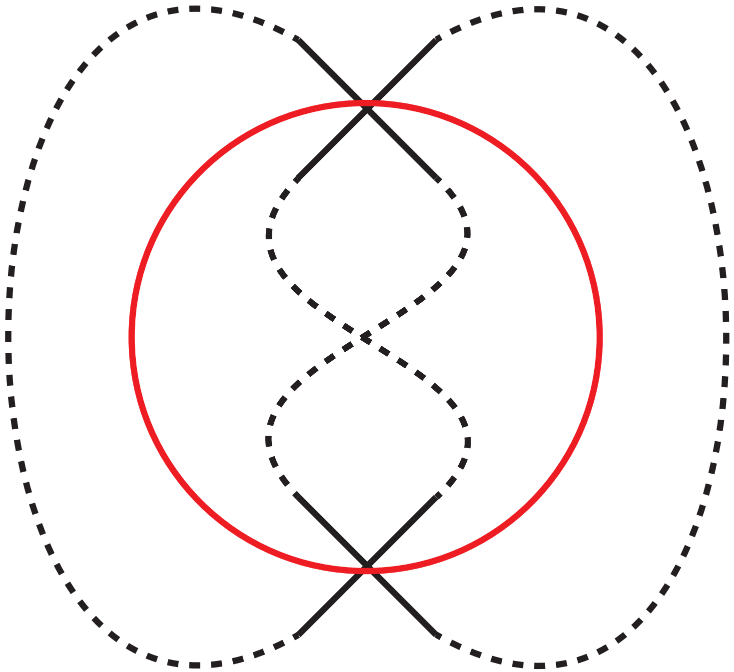


<結果(1) [I.-Takimura],証明に入ります>

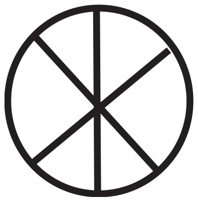
P: reduced.

r(P)=1

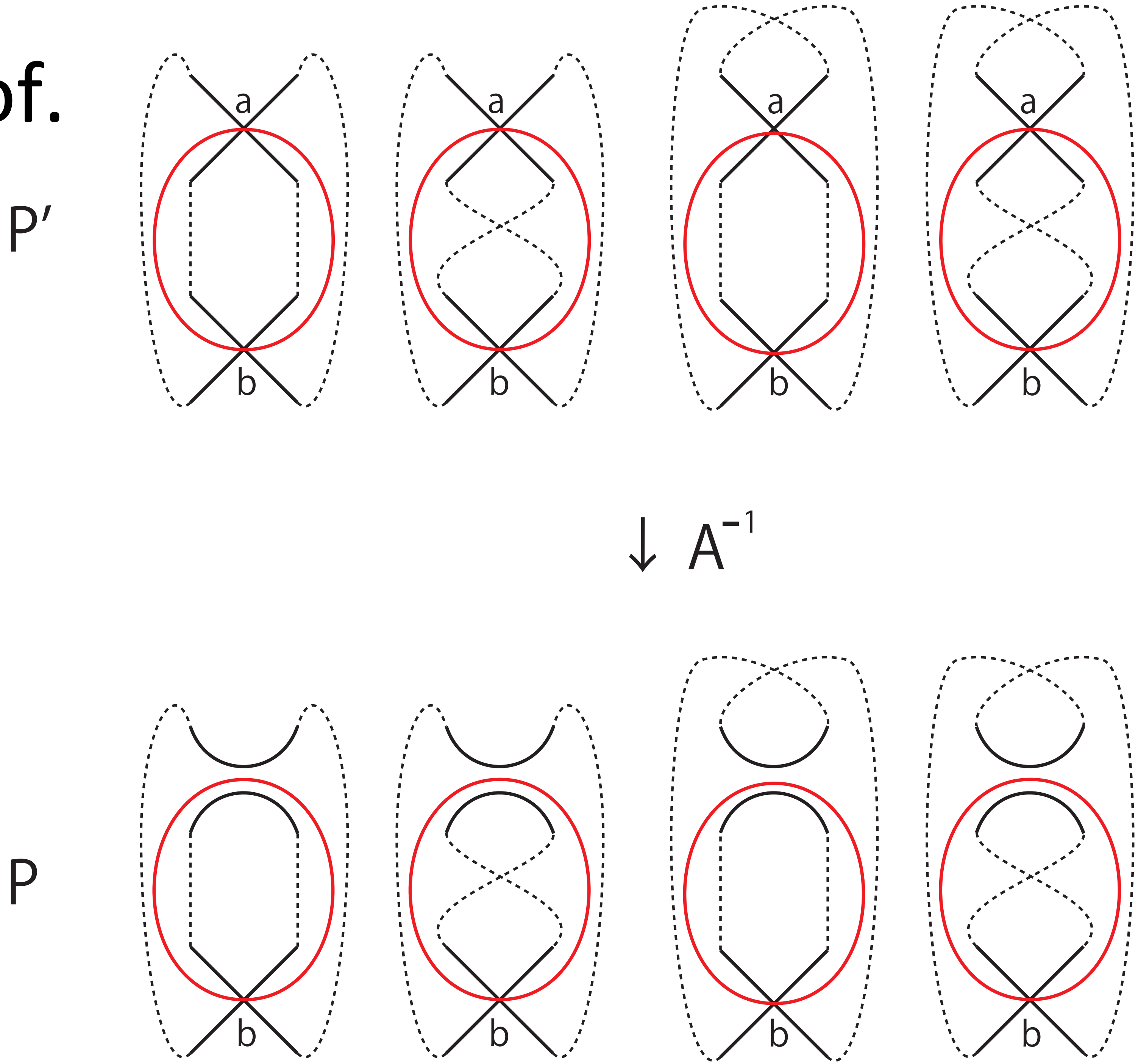
⇔図のような、ちょうど2交点と交わるsimple circleがとれる.



Cor.

r(P)=1⇒Pは  をもつ

Proof.



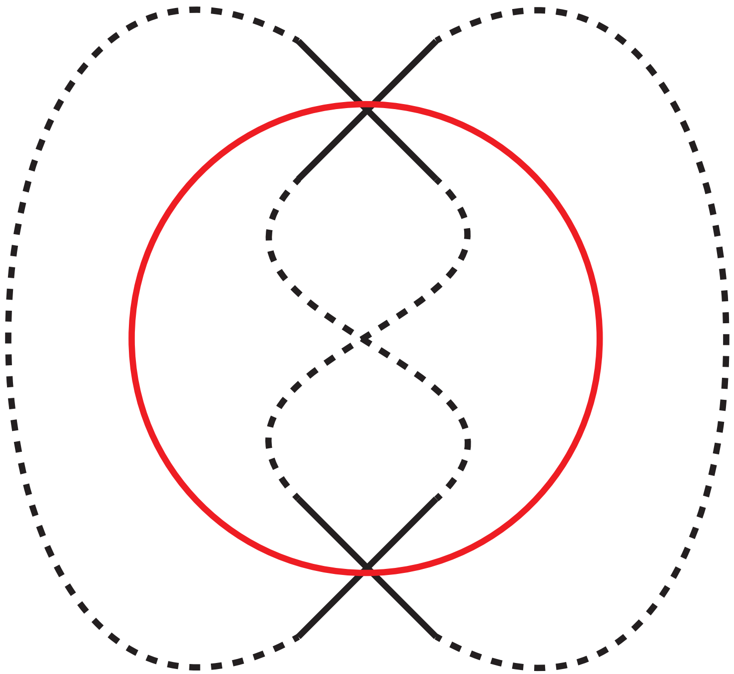
□

結果(1) [I.-Takimura]

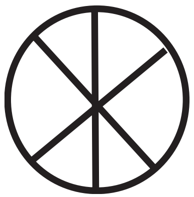
P: reduced.

r(P)=1

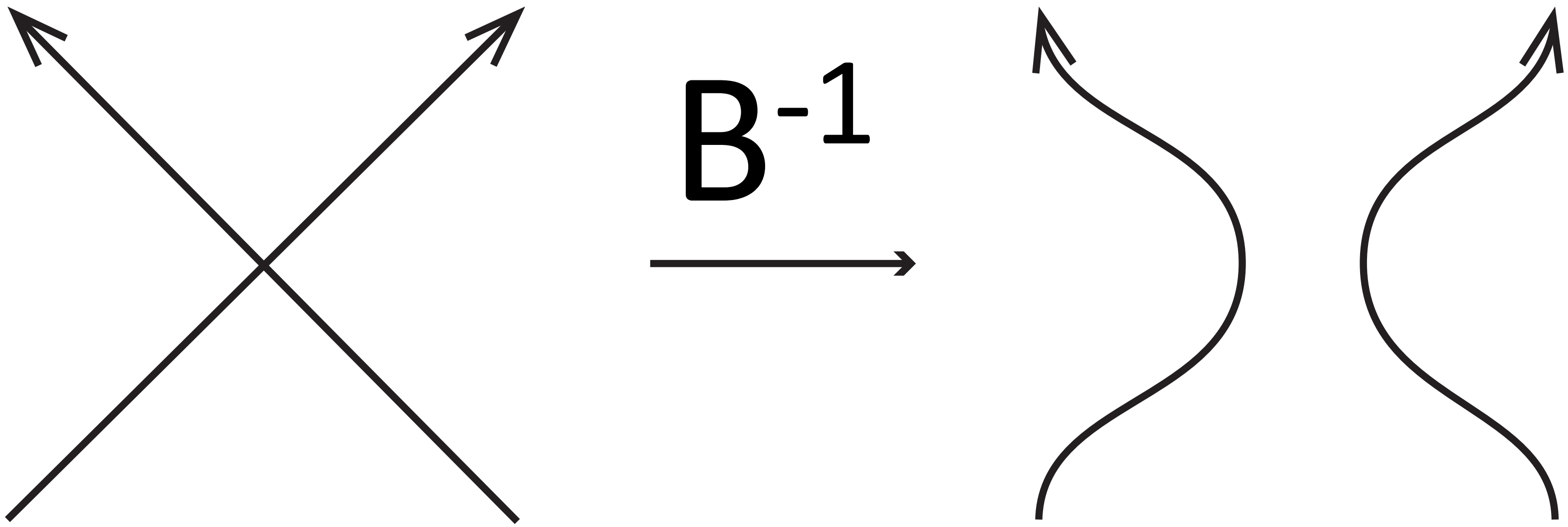
⇔図のような、ちょうど2交点と交わるsimple circleがとれる.



Cor.

r(P)=1⇒Pは  をもつ

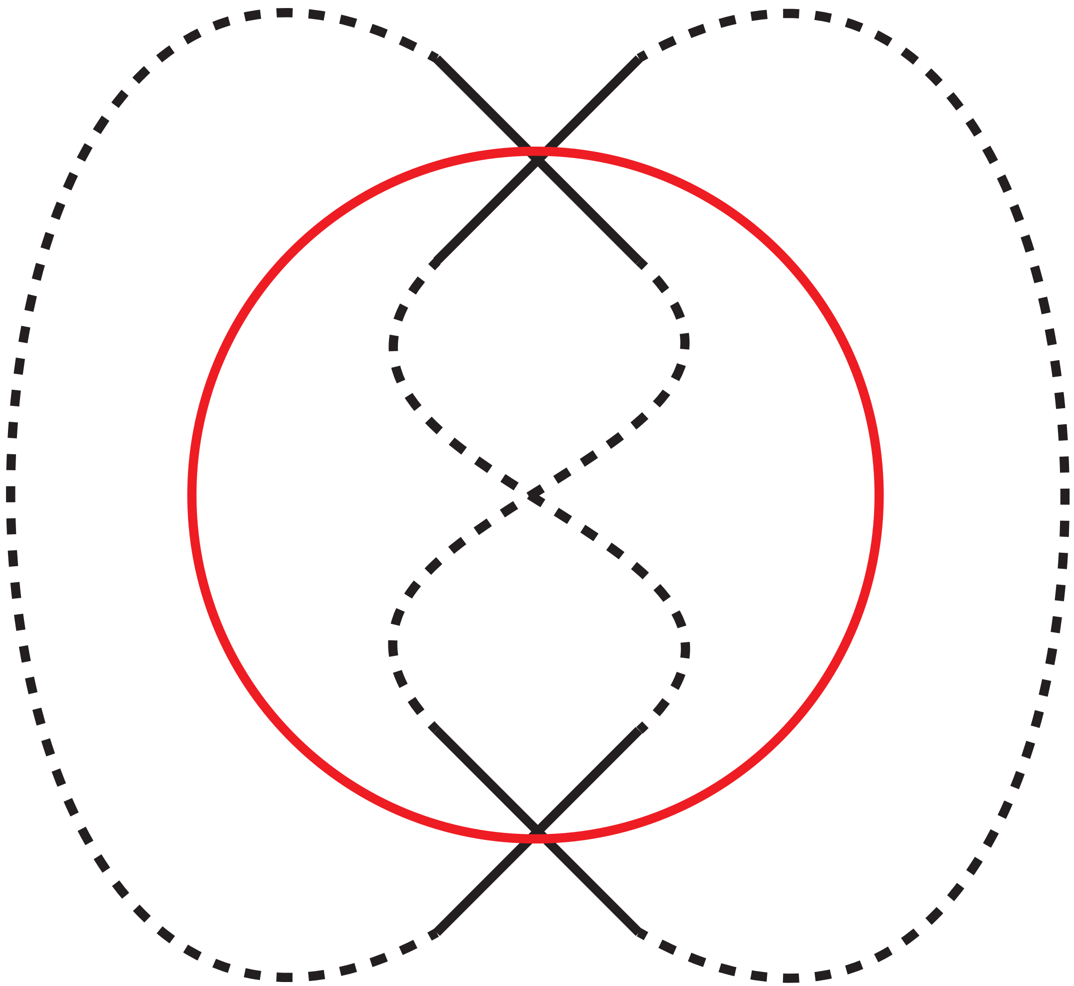
結果(3)[I.-Takimura]<下からの評価>



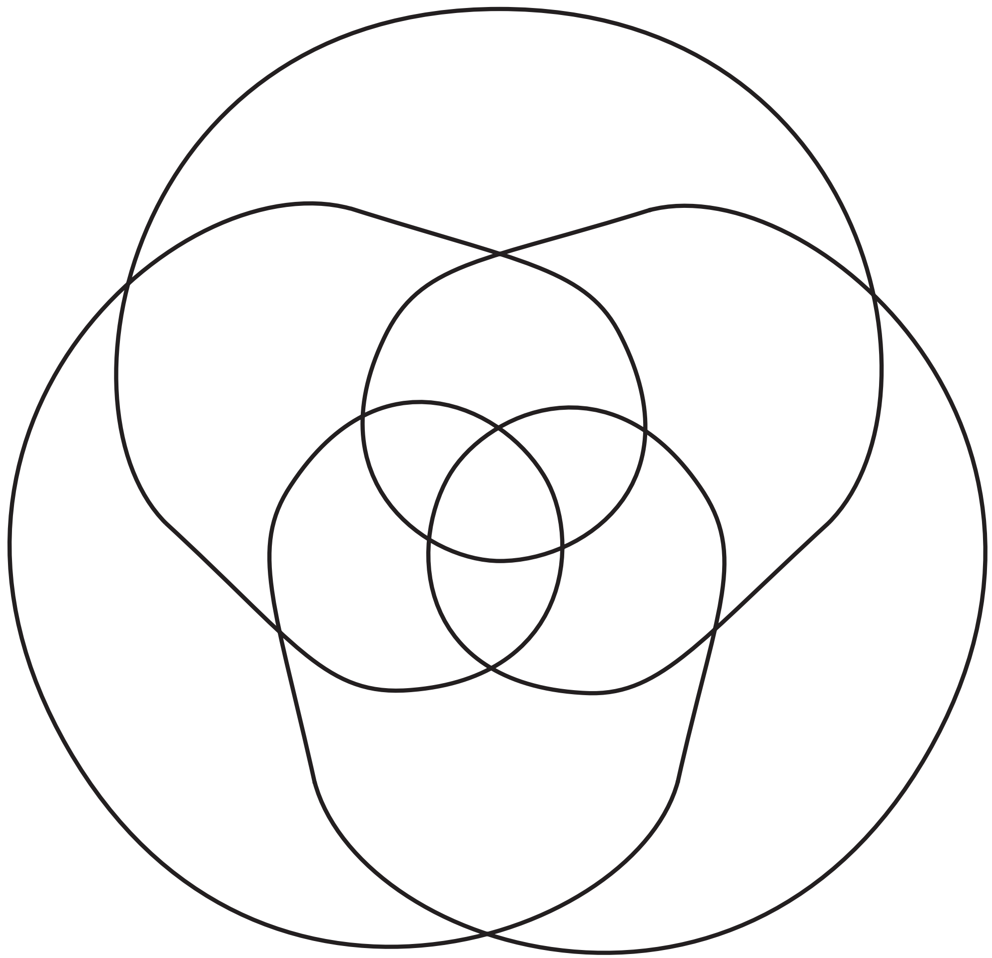
$|\tau(P)|$: すべての交点をA^{-1}でsmoothingした後のcircleの個数.

$$|\tau(P)| = 1 \Rightarrow 2 \leqq r(P).$$

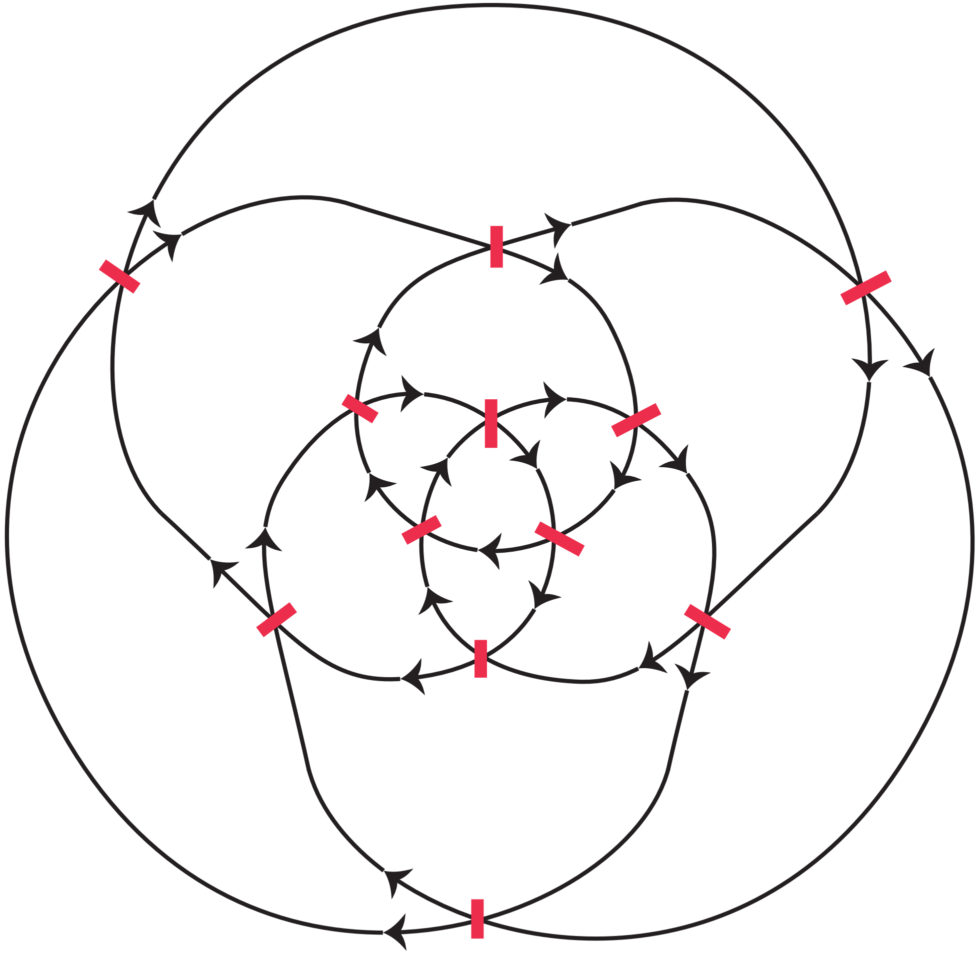
Proof. $r(P) = 1 \Rightarrow 1 \neq |\tau(P)|$.



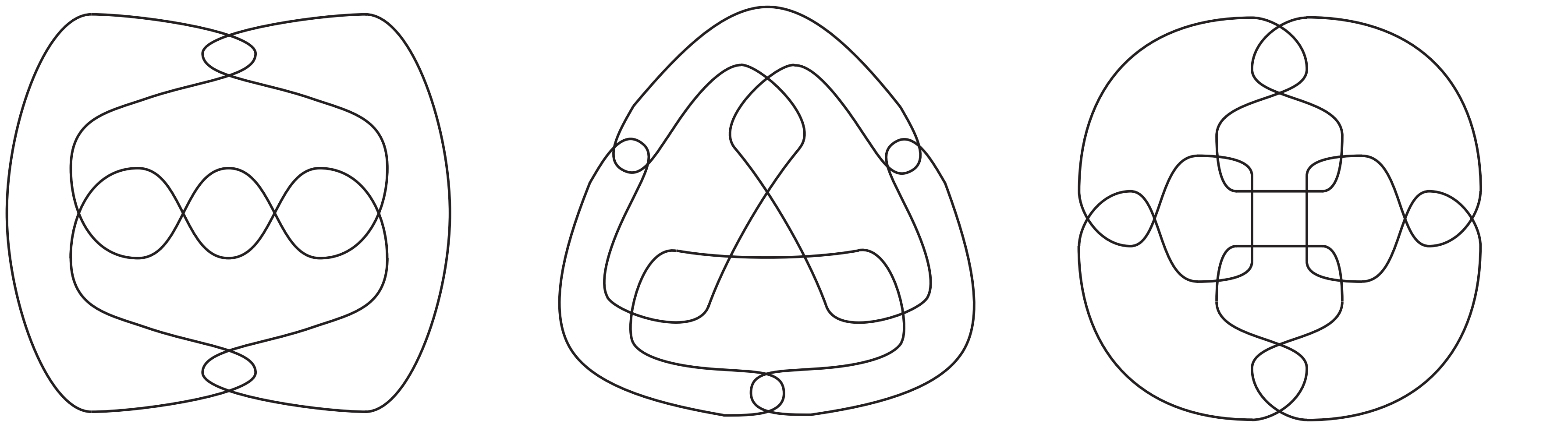
Example 1



Example1 $|\tau(P)|=1 \Rightarrow 2 \leqq r(P)$



Example 2 $|\tau(P)|=1 \Rightarrow 2 \leqq r(P)$



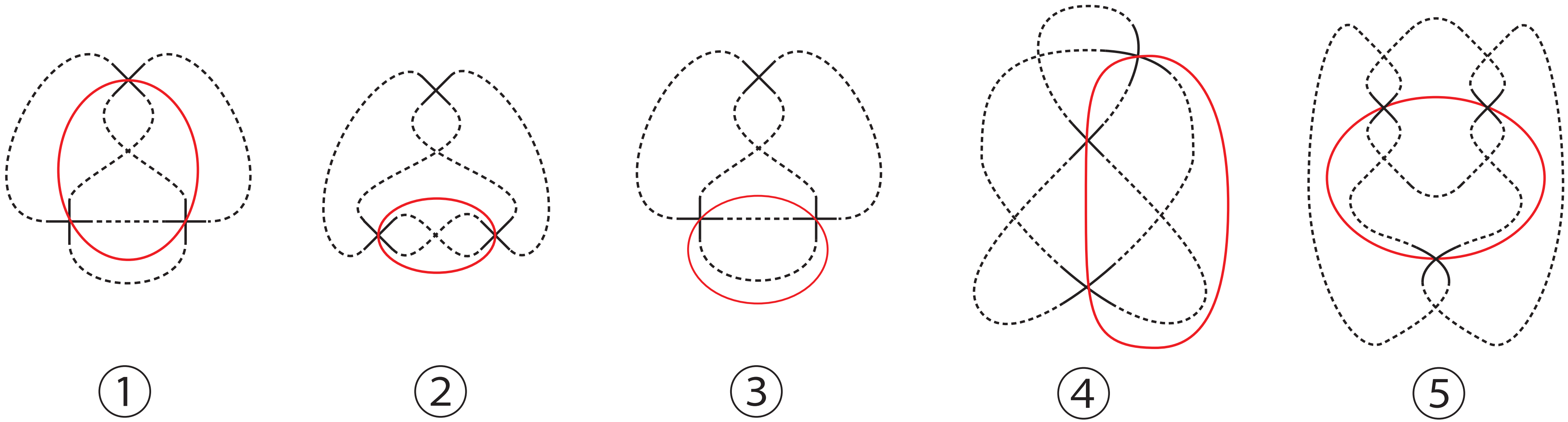
…

結果(2)[I.-Takimura, Topology Appl.2015]

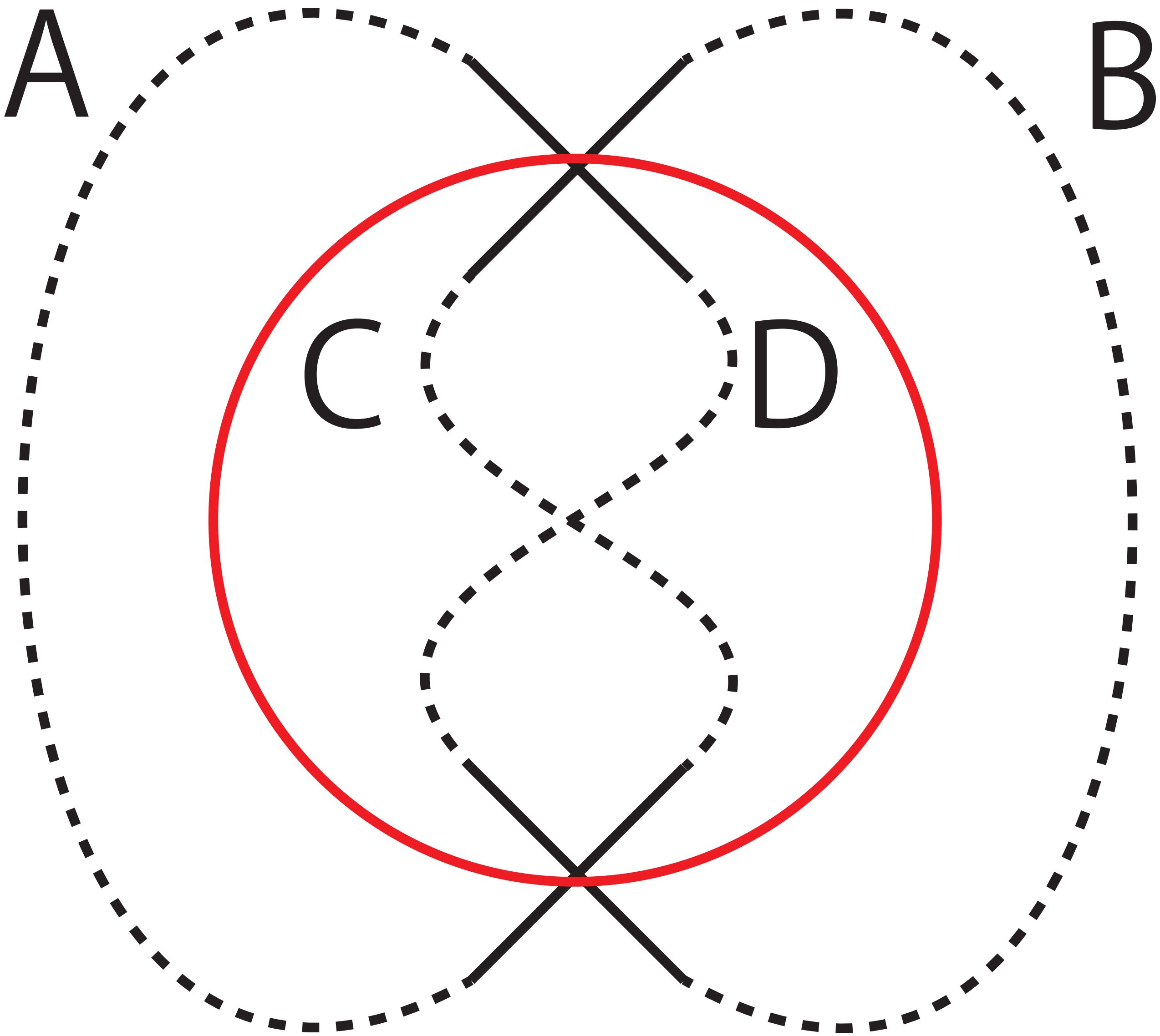
P: reduced

r(P)=2

⇔Pと2or3交点で交わる下記の円周が存在かつr(P)≠1.



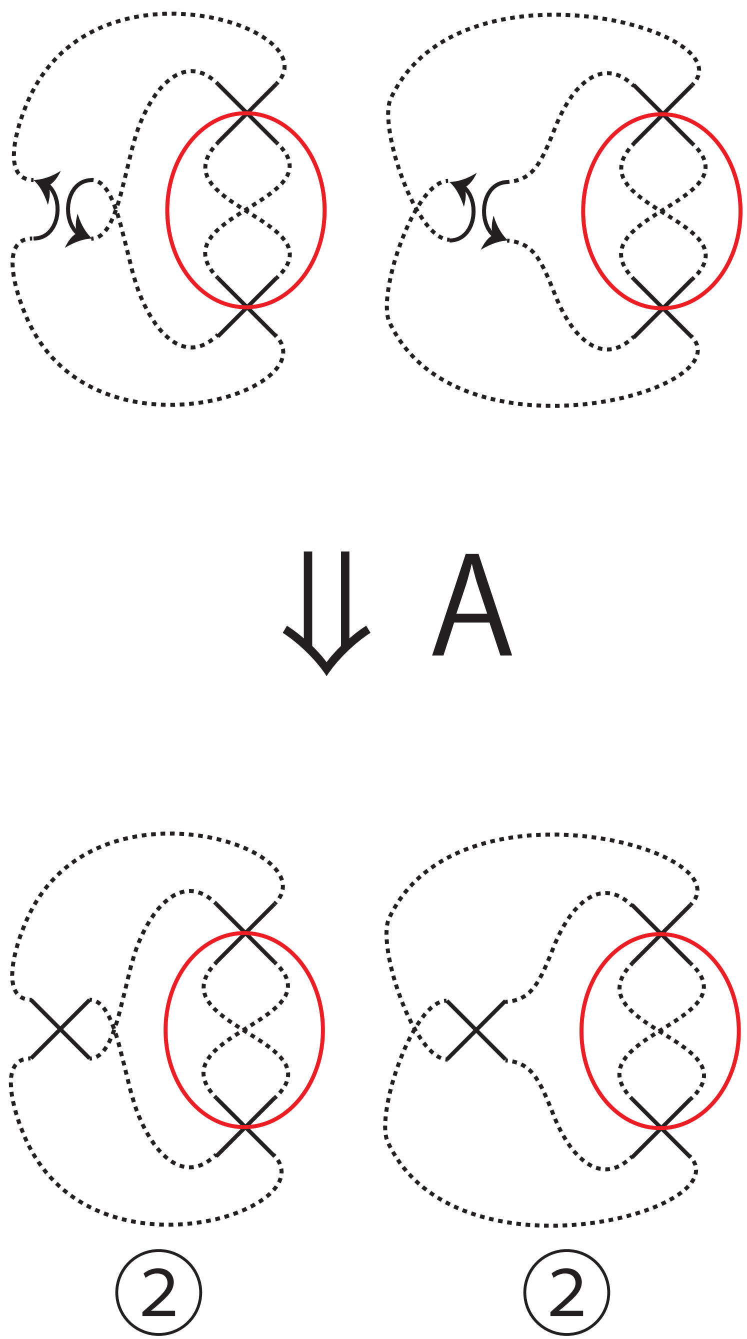
Proof



Proof

Case 1

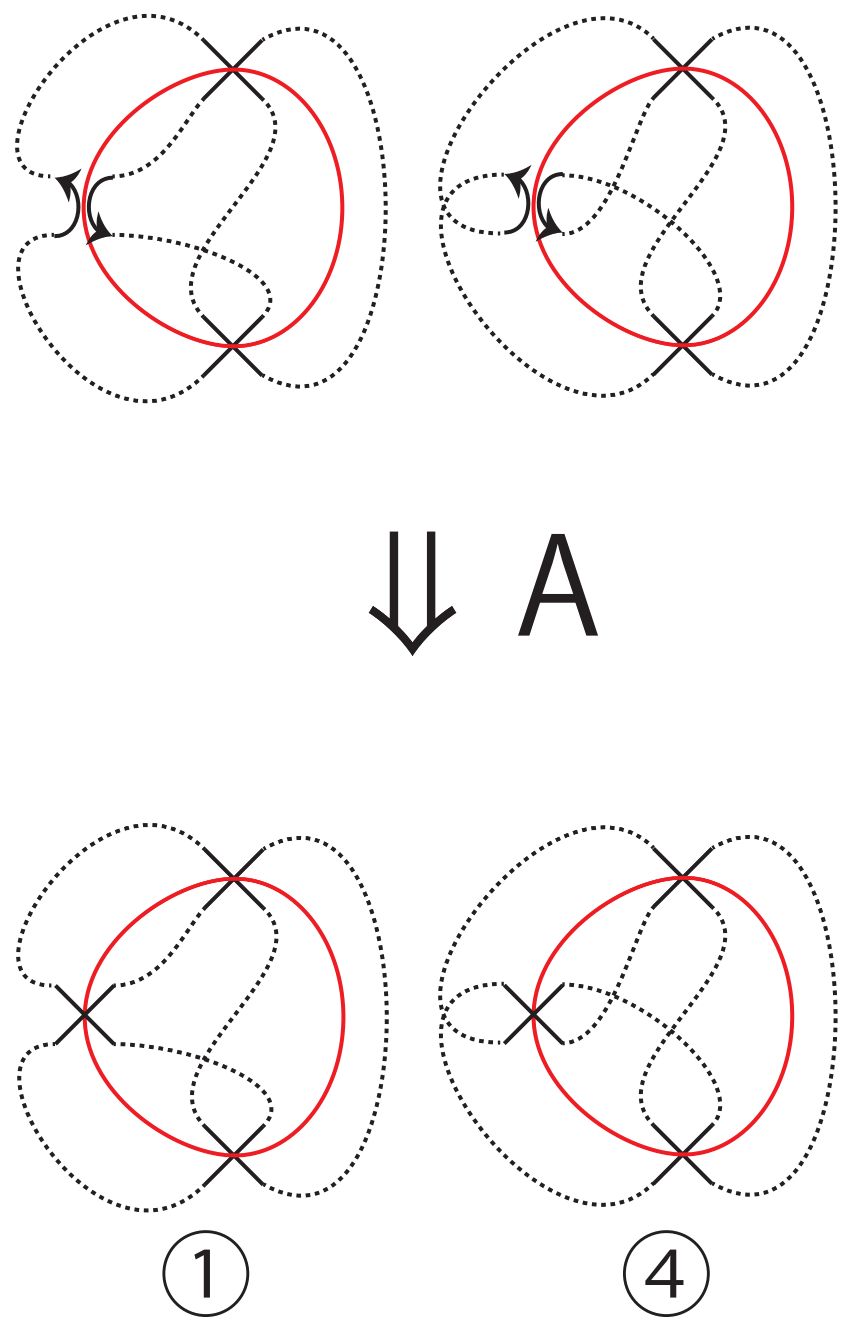
(A, B)



Proof

Case 2a

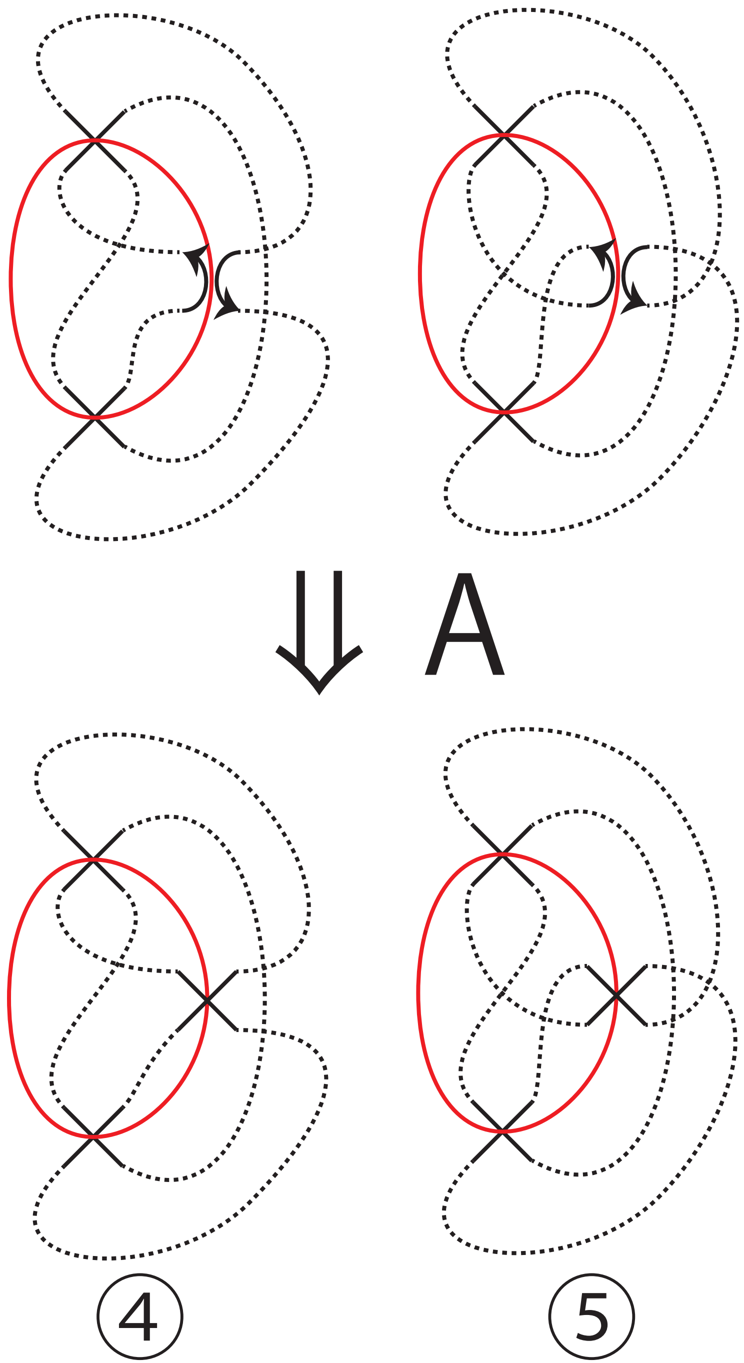
(A, C)



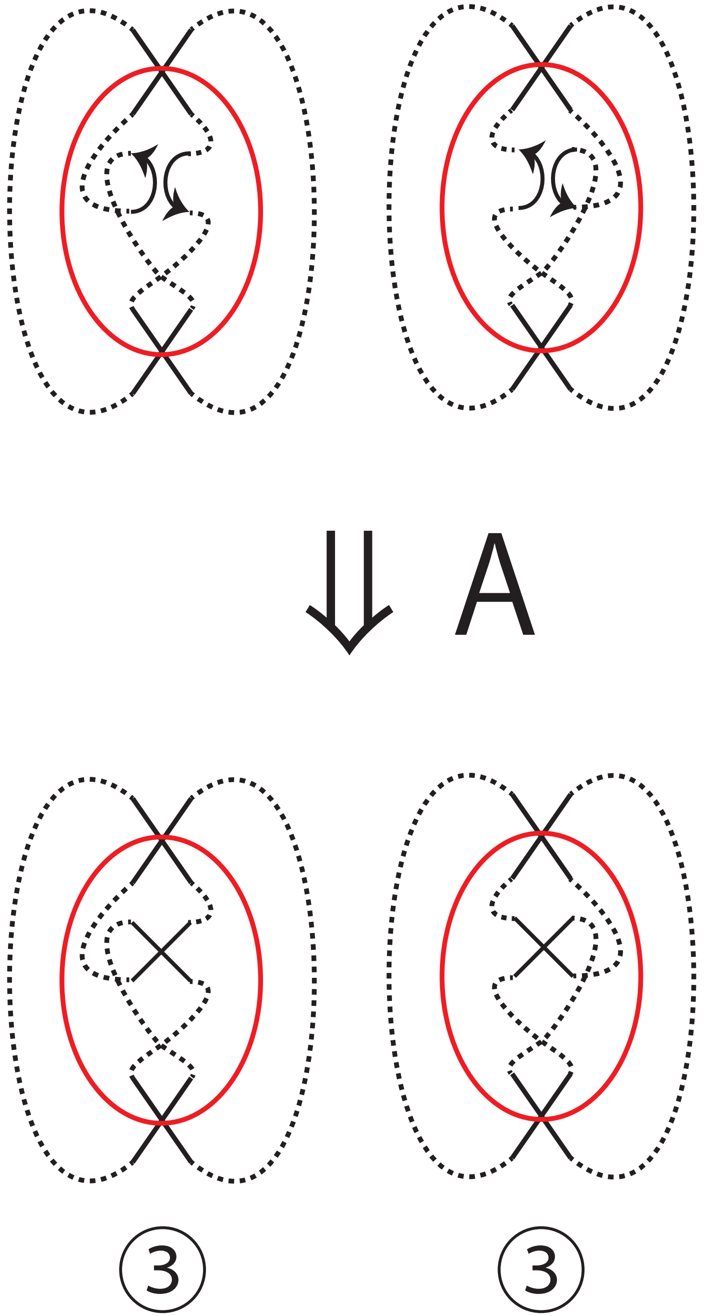
Proof

Case 2b

(A, C)



Proof
Case 3
(C, D)

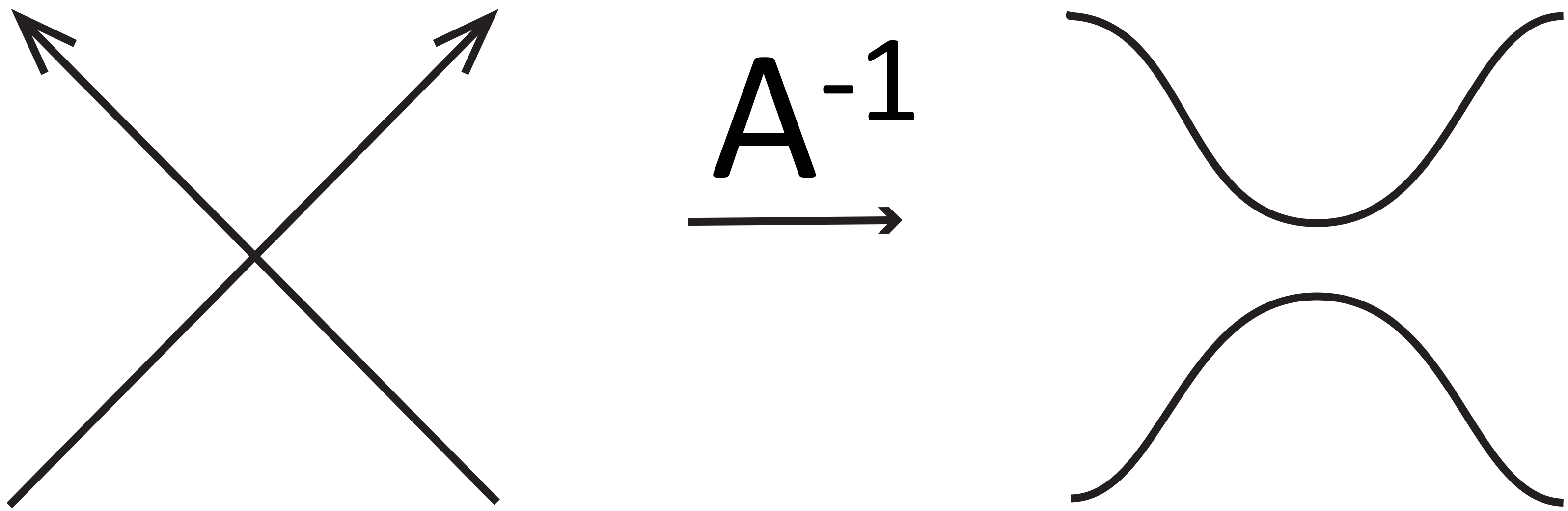


□

Definition 既約度(Reductivity)[清水]

P: knot projection, 既約度 r(P)とは

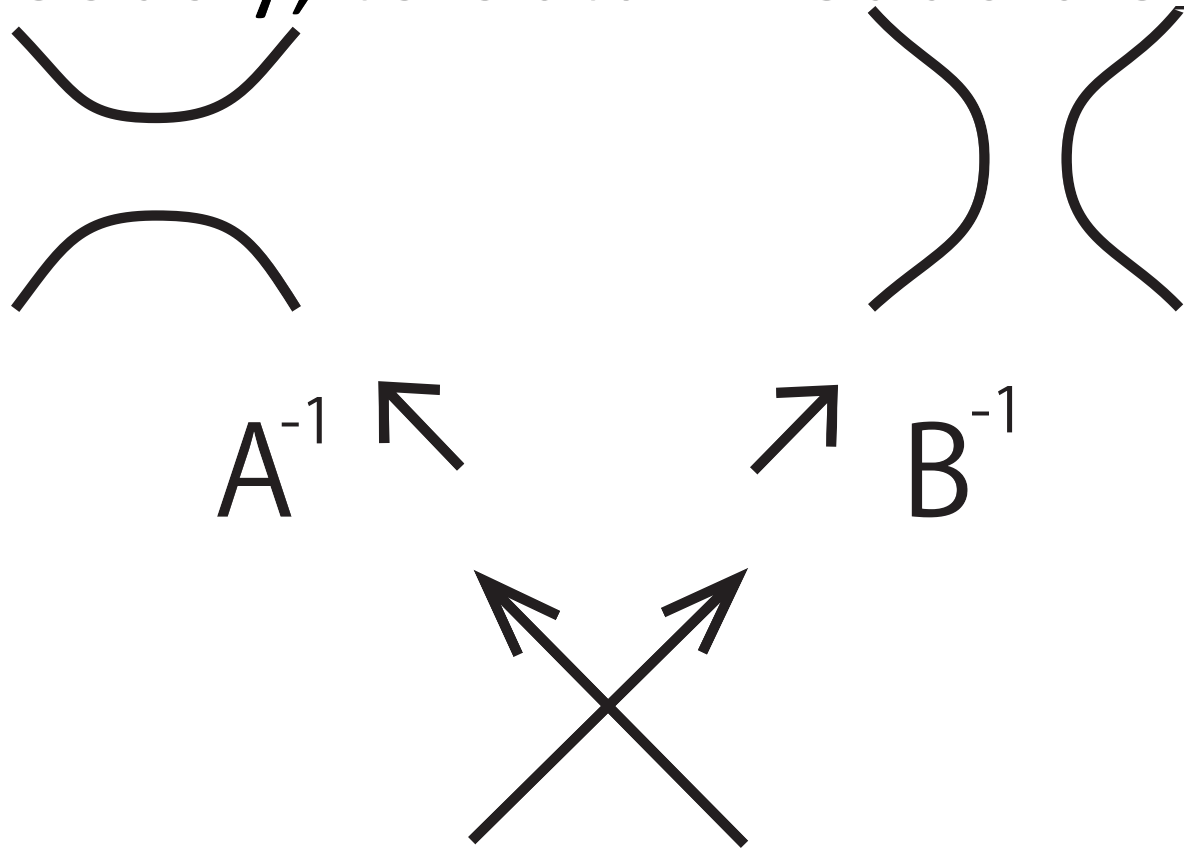
min{ Pをreducible P'にさせるA^{-1}の回数}.



Definition 既約度 t(P)

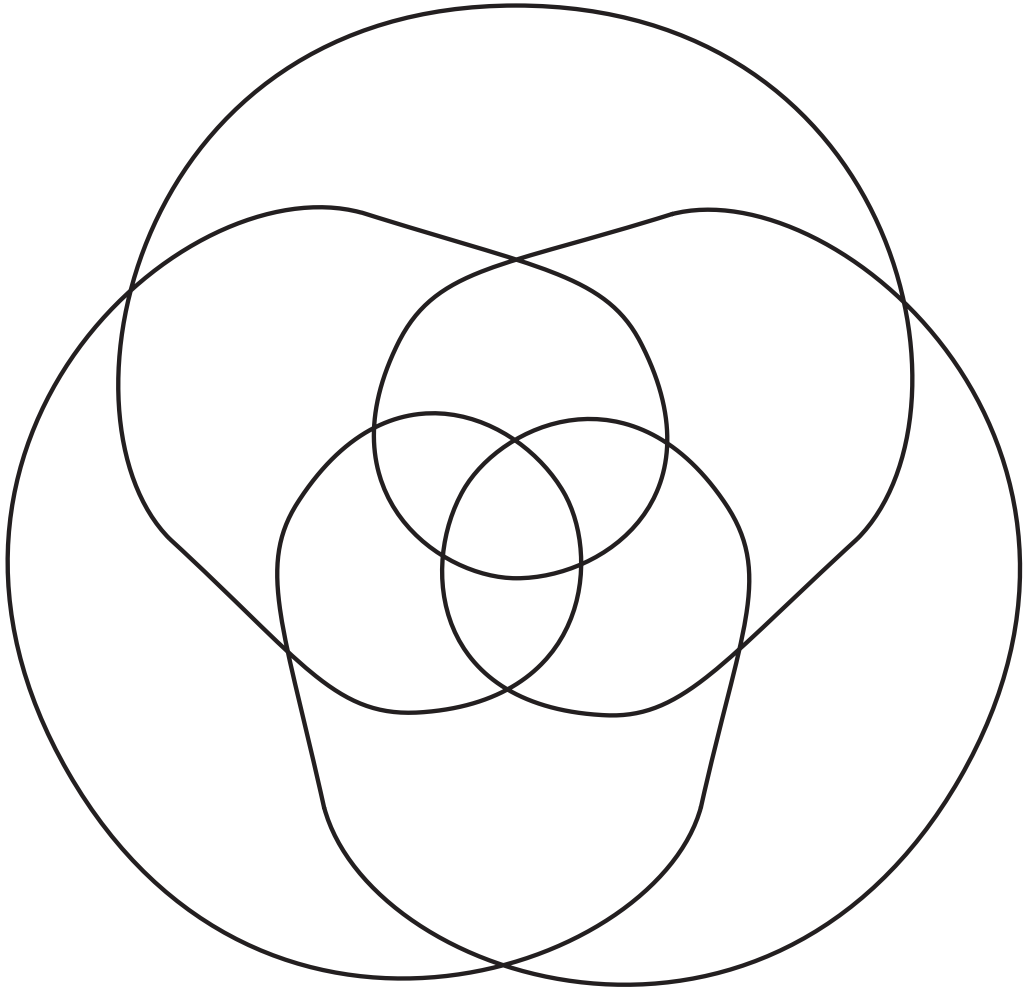
P: knot projection.

t(P)=min{ number of B^{-1} or A^{-1} ,applied simultaneously, to obtain reducible P'}.

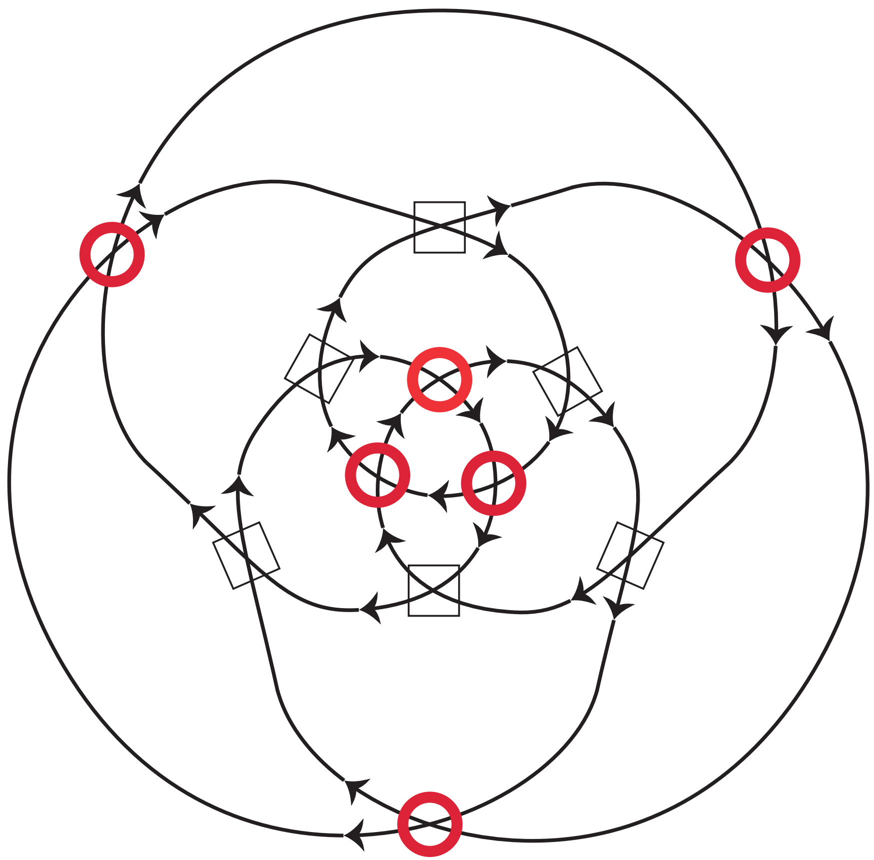


Example[Taniyama, Barthel]

$t(P)=2$ and $r(P)=3$

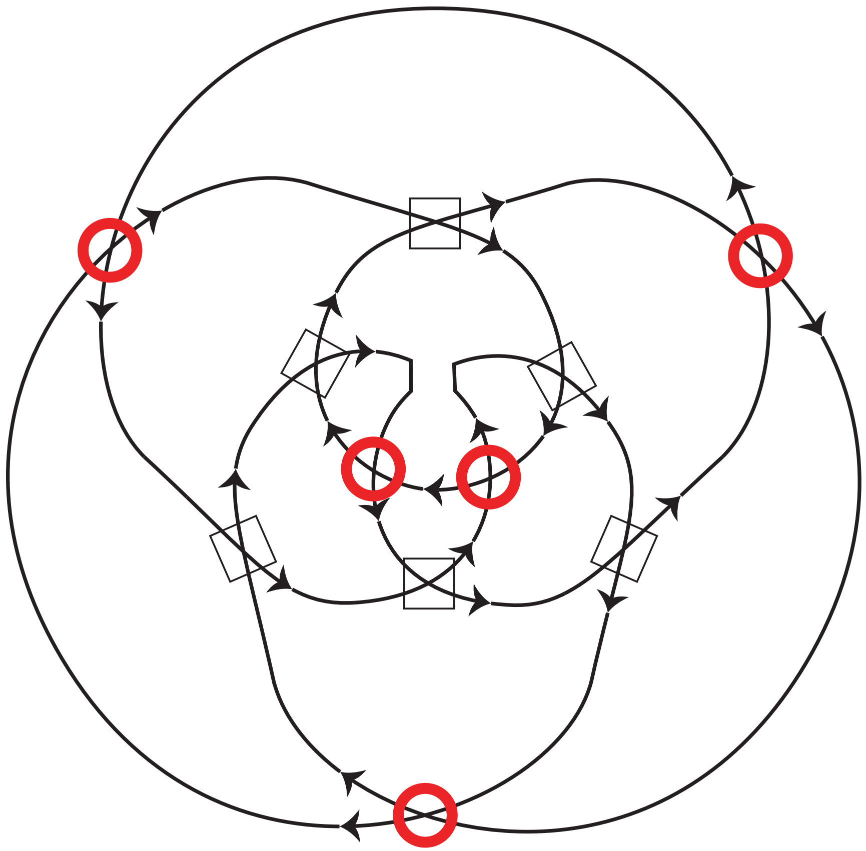


$t(P) \leqq 2,\ r(P) \leqq 3$



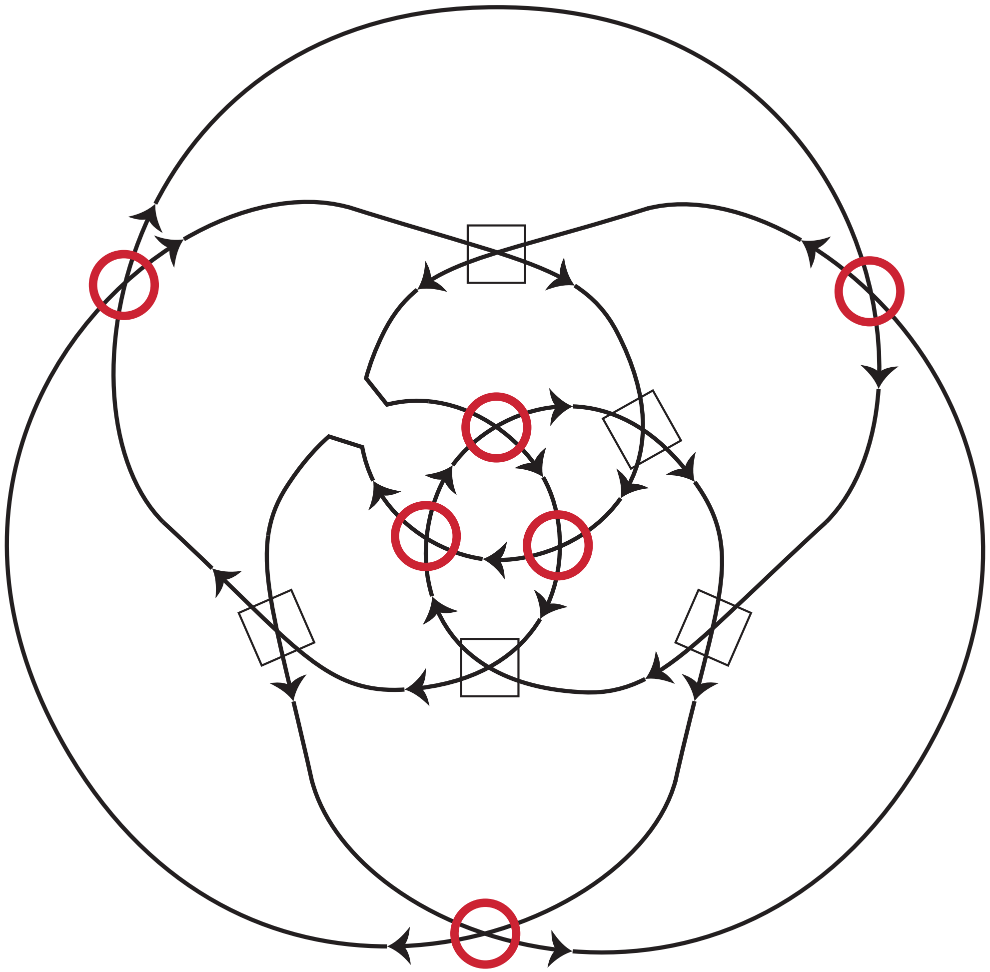
(1)

$t(P) \leqq 2,\ r(P) \leqq 3$



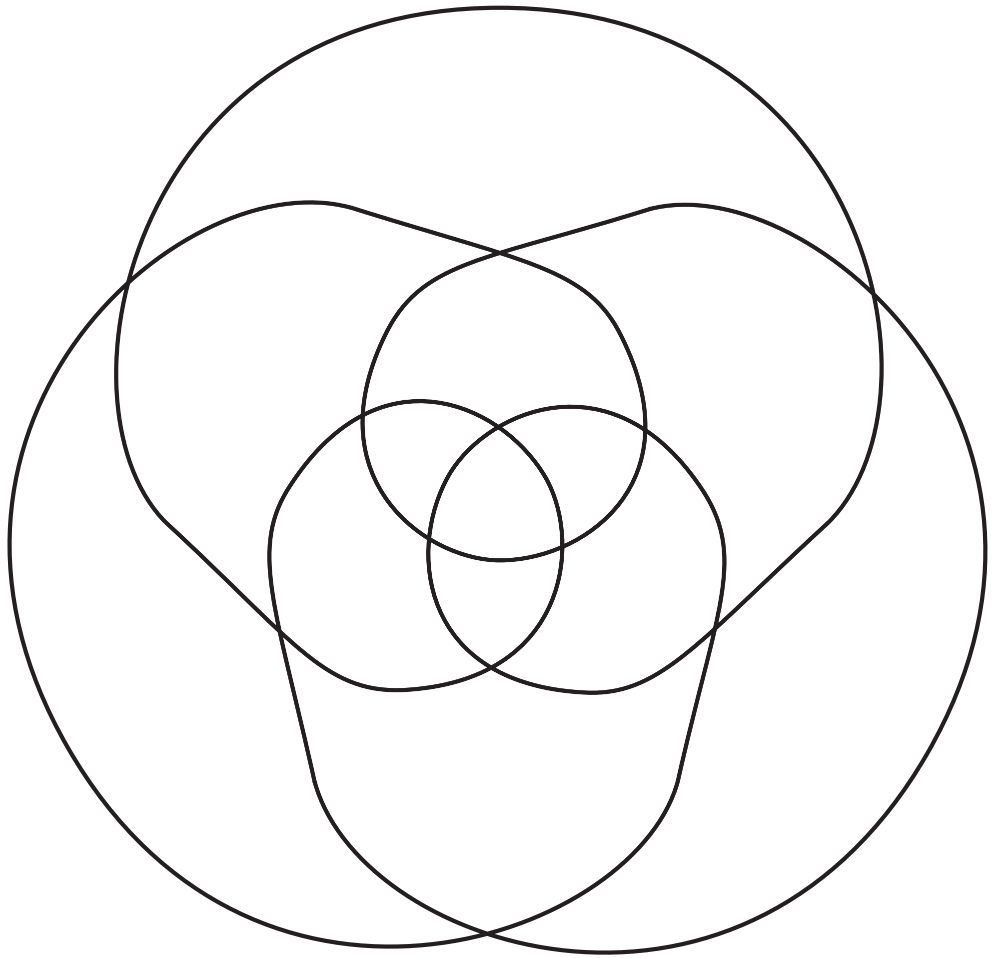
(2)

$t(P) \leqq 2,\ r(P) \leqq 3$



(3)

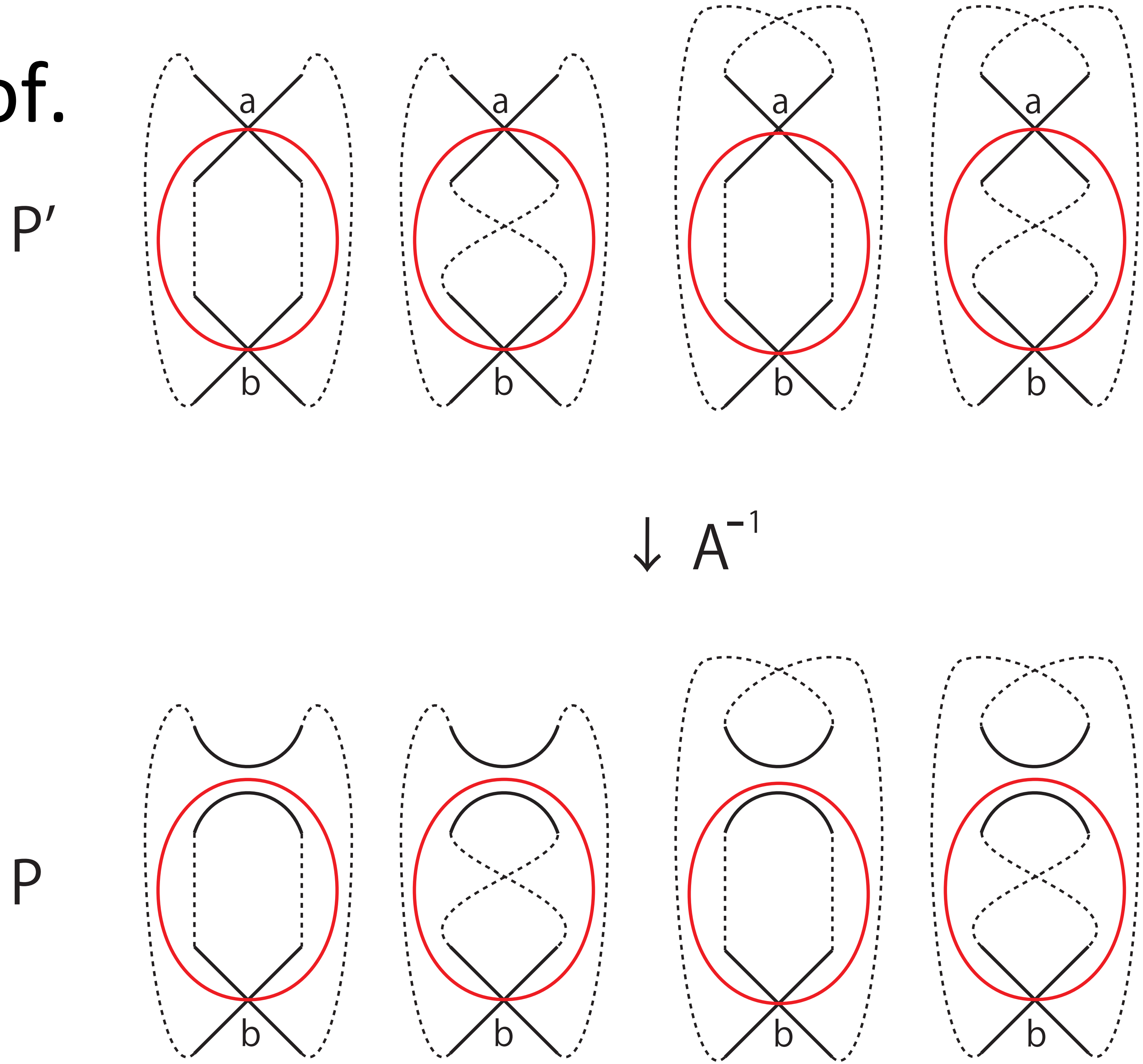
Prop.[I.-Takimura], $|\tau(P)|=1 \Rightarrow 2 \leqq t(P)$
を使っても良い



(結果4) [I. -Takimura]

$$r(P)=1 \Leftrightarrow t(P)=1.$$

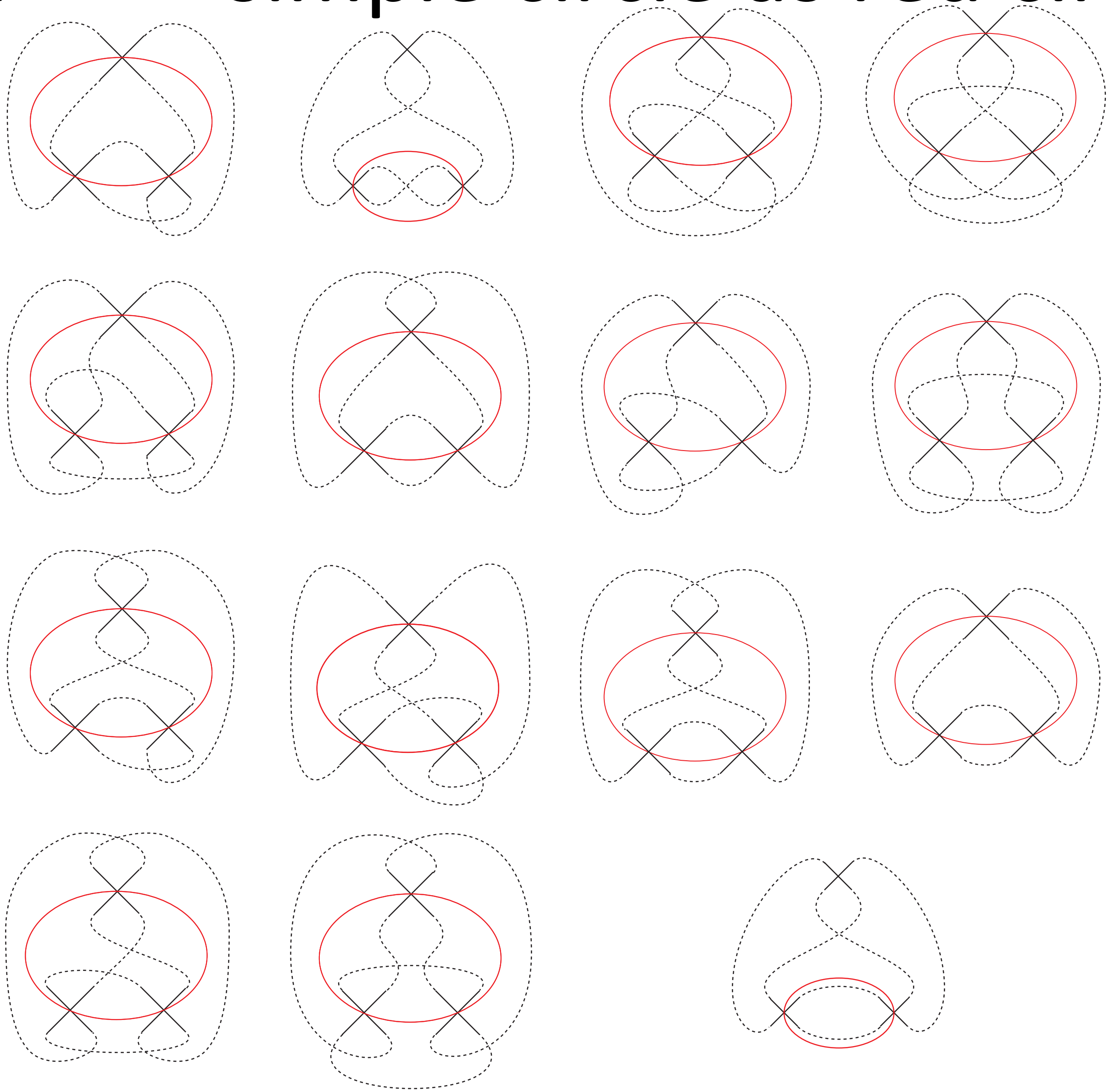
Proof.



□

結果(5) [I.-Takimura]

t(P)=2 ⇔ ∃simple circle as red circle.



[proof]・Step 1. Find P with y(P)=2

y(P) = min{＃A^{-1}, applied simultaneously, to obtain reducible P'}

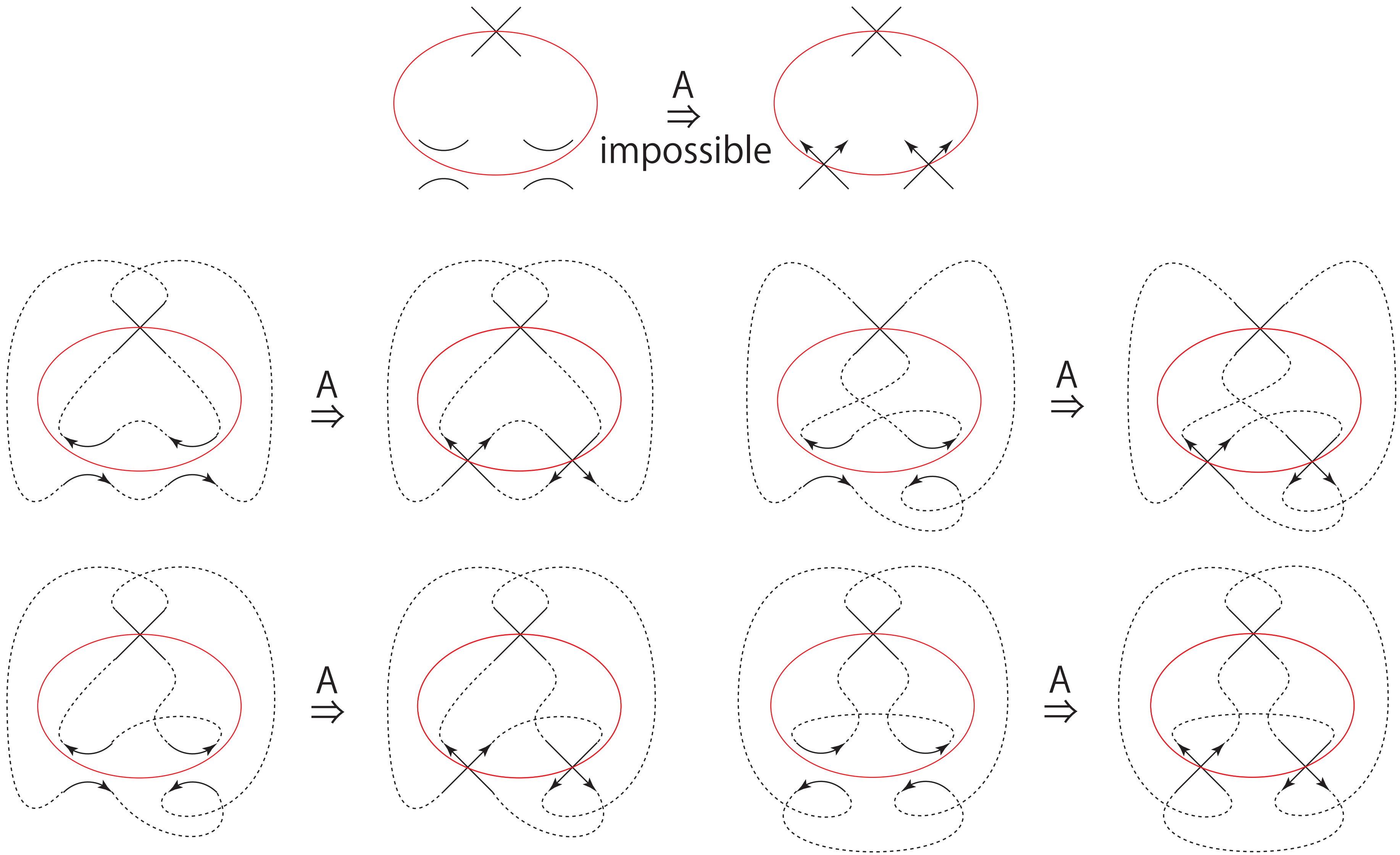
・Step 2. Find P with i(P)=2.

i(P)= min{＃B^{-1} to obtain reducible P'}

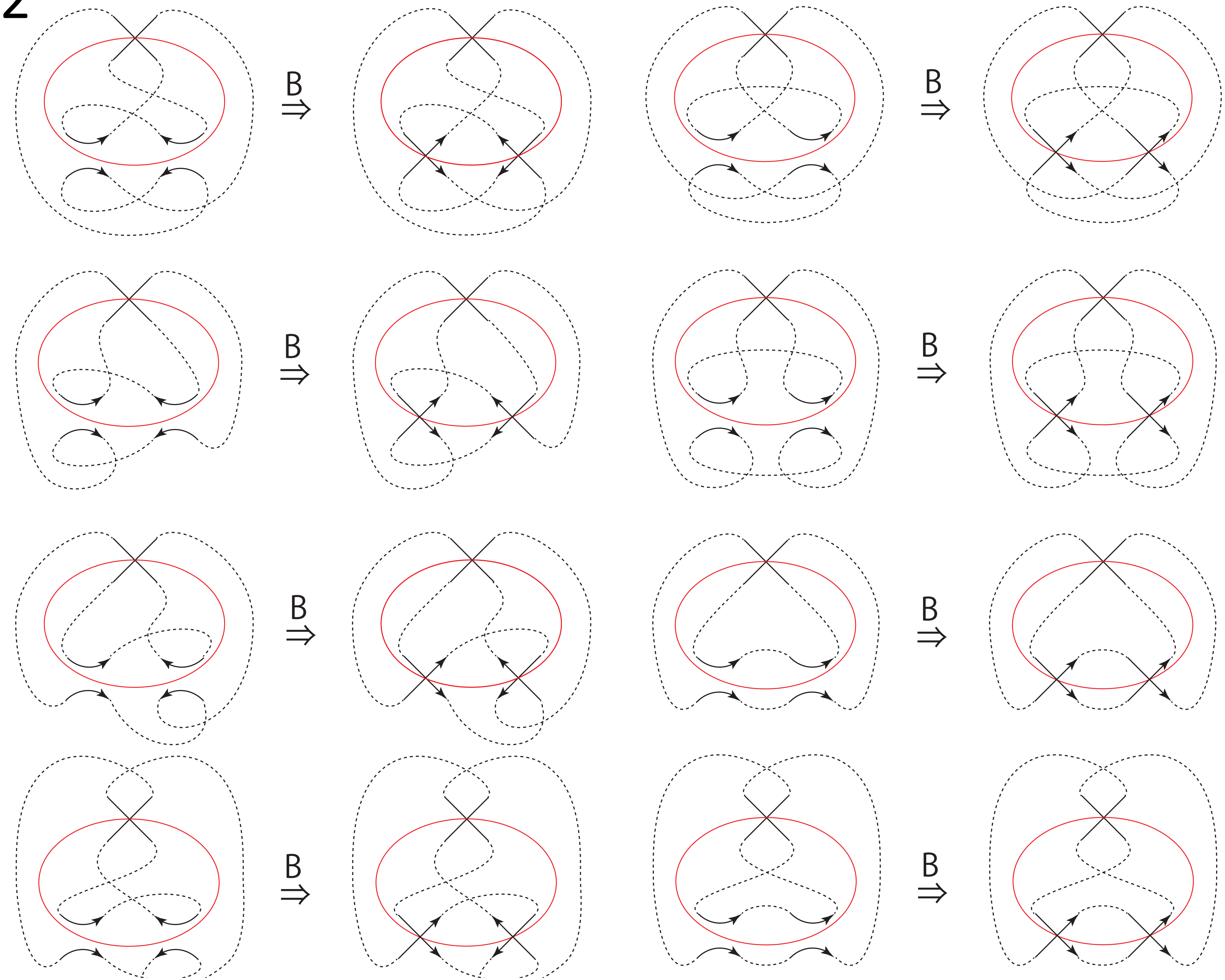
・Step 3. Find P with t(P)=2.

t(P)= min {＃A^{-1}, or B^{-1} applied simultaneously, to obtain reducible P'}

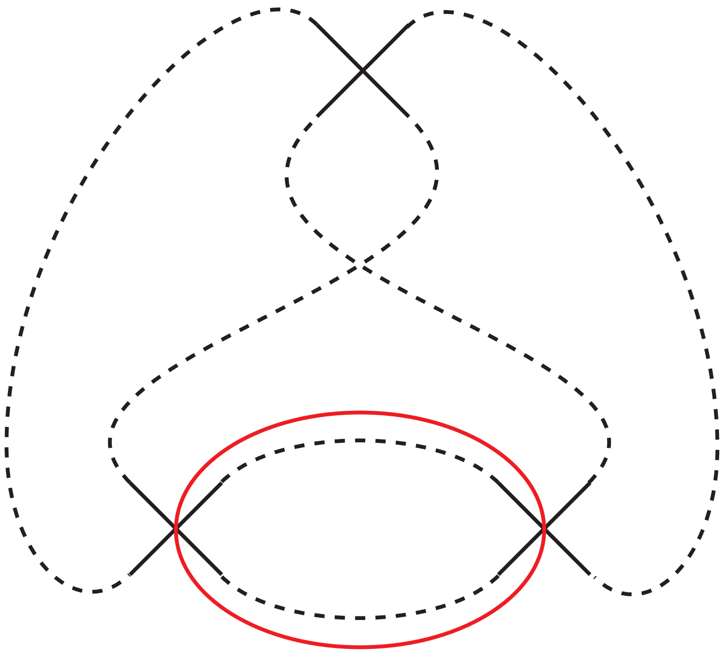
Proof (Step 1)

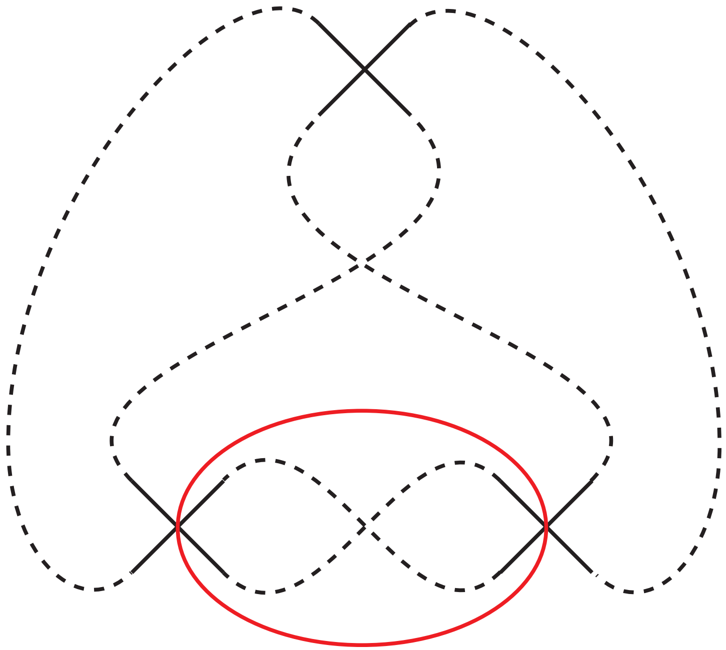


Step 2

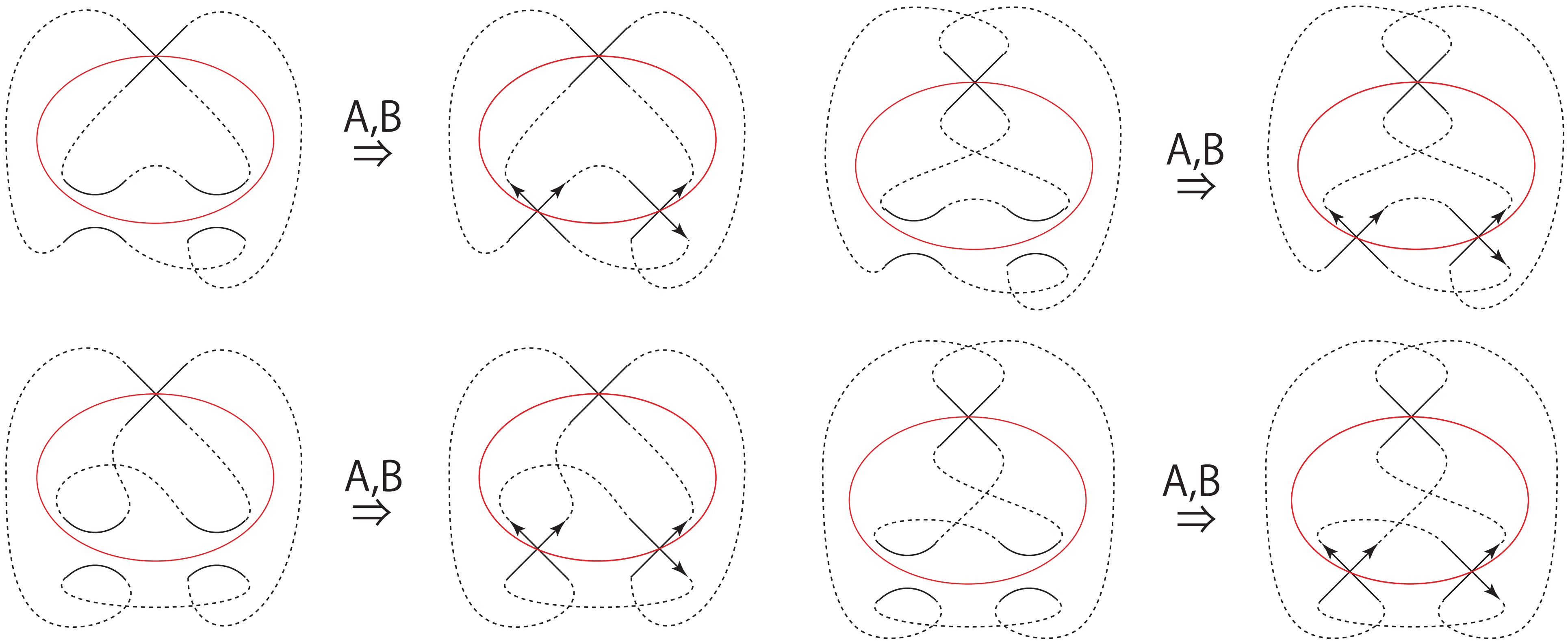


Case 2, two exceptions





Proof (Step 3)



□

では、$3 \leqq t(P)$ なるPは何か？

Open problem

$\exists P,\ t(P) = 3 \text{ or } 4$ ？

では、3 ≦　t(P)なるPは何か？

Open problem

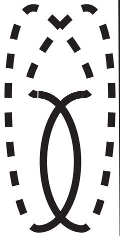
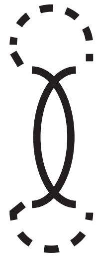
∃P, t(P) = 3 or 4 ?

cf. ∃P, r(P)=3

Open problem [清水] ∃P, r(P)=4?

Fact. t(P) ≦ r(P) ≦ 4.

Thank you for your listening
(↓upper bounds, blank: open)

			A 3-gon	B 3-gon	C 3-gon	D 3-gon
r	1	2	2	3	3	4
t	1	2	2	3	2	4
y	1		2	**3**		
i	∞	2			2	