Title Linearly Free Graphs

Abstract An embedding of a graph into the Euclidean3-space $\mathbb{R}^3$ is said to be *linear*, if it sends every edge to be a line segment. And we say that an embedding f of a graph G into $\mathbb{R}^3$ is *free*, if $\pi_1(\mathbb{R}^3 - f(G))$ is a free group. Lastly a simple connected graph is said to be *linearly free* if every its linear embedding is free. In 1980s it was proved that every complete graph is linearly free, by Nicholson.

In this talk, we develop the Nicholson's arguments into a general notion, and establish a sufficient condition for a linear embedding to be free. As an application of the condition we give a partial answer for a question: how much can the complete graph K_n be enlarged so that the linear freeness is preserved and the clique number dose not increase? And an example supporting our answer is provided.

As the second application it is shown that a simple connected graph of minimal valency at least 3 is linearly free, if it has less than 8 vertices. The conditional inequality is sharp, because there is a graph with 8 vertices which is not linearly free. It is also proved that for $n, m \leq 6$ the complete bipartite graph $K_{n,m}$ is linearly free.