On Miyazaki's fibered, negative amphicheiral knots

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Amphicheiral knots

Definition

A knot K in S^3 is $-$*amphicheiral* if there exists an orientation reversing homeomorphism $f\colon (S^3, K) \to (S^3, K)$ such that $f(K)$ is K with the reversed orientation. A knot K in S^3 is *strongly* $-$*amphicheiral* if we can choose f to be an involution.

Example

The figure-eight knot, 4_1, is strongly $-$amphicheiral.

- **Kawauchi**: Every hyperbolic $-$amphicheiral knot is strongly $-$amphicheiral.
- **Hartley, Whitten**: Not every $-$amphicheiral knot is strongly $-$amphicheiral.

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It is a Miyazaki knot.

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Motivations

This is a joint work with Min Hoon Kim (arXiv:1604.04870).
The original motivation is the *slice-ribbon conjecture*.

- A knot $K \subset S^3$ is called *slice* if it bounds an embedded disk D in D^4, and it is called *ribbon* if it bounds an immersed disk in S^3 with only ribbon singularities.
- Every ribbon knot is a slice knot. The slice-ribbon conjecture asks if the converse is true.

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We are interested in Miyazaki knots because:

- **Miyazaki**: For a Miyazaki knot K, the cable knot $K_{2n,1}$ is algebraically slice but not ribbon.
- **Kawauchi**: For a strongly $-$amphicheiral Miyazaki knot K, the cable knot $K_{2n,1}$ is rationally slice.
- If we can find a certain $K_{2n,1}$ that is slice, then this gives a counterexample to the slice-ribbon conjecture.

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Classification of (Miyazaki) knots

Remember that a Miyazaki knot is a fibered, $-$amphicheiral knot with irreducible Alexander polynomial.

Fact: Every knot is either hyperbolic, a torus knot, or a satellite knot.

- By Kawauchi, a hyperbolic Miyazaki knot is strongly $-$amphicheiral.
- Non-trivial torus knots are not $-$amphicheiral, hence not Miyazaki.
- Hence, we should focus on satellite Miyazaki knots.

Let $K = P(J)$ denote a satellite knot K with pattern $(S^1 \times D^2, P)$ and companion J.

Lemma (Kim-W)

If a satellite knot $K = P(J)$ is Miyazaki, then P is an unknotted pattern, J is Miyazaki, and $g(J) < g(K)$.

Consequently, all Miyazaki knots can be obtained from hyperbolic ones via iterated satellite operations.

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Proof of the lemma

Main observation: the fiberness, chirality, and the Alexander polynomial of a satellite knot $K = P(J)$ are "determined" by those of $(S^1 \times D^2, P)$ and J.

- $\Delta_K(t) = \Delta_P(t)\Delta_J(t^w)$, where w is the winding number of P.
- $K = P(J)$ is fibered if and only if both $(S^1 \times D^2, P)$ and J are fibered. Moreover, the winding number $w \neq 0$.

Hence, if K is Miyazaki, then both P and J are fibered.

Since P and J are fibered, $\deg \Delta_P(t) = 2g(P)$ and $\deg \Delta_J(t) = 2g(J)$. From the irreducibility of $\Delta_K(t)$ and $\Delta_J(t^w) \neq 1$, we conclude that $\Delta_P(t) = 1$ and hence P is an unknotted pattern.

Since $\Delta_K(t)$ is irreducible, $\Delta_J(t^w) = \Delta_K(t)$ implies that $\Delta_J(t)$ is also irreducible. Actually $|w| \geq 2$, so $g(K) = |w|g(J) > g(J)$.

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Symmetry of links

In order to study the chirality of a satellite knot, we introduce the more general notion of symmetry of a link.

We say that a link $(S^3, L = K_1 \sqcup \cdots \sqcup K_n)$ *has symmetry* $(\alpha, \epsilon_1, \cdots, \epsilon_n)$ if there exists a self-homeomorphism f of S^3 of class α that restricts to a self-homeomorphism of each component K_i of class ϵ_i for each i.

- α takes the value ± 1 or $J_\pm$, which stands for orientation preserving/reversing homeomorphisms or involutions of S^3, respectively
- $\epsilon_i = \pm 1$ depending on whether $f|_{K_i}$ preserves or reverses homeomorphisms of K_i.

In particular, a knot is (S^3, K) is $-$amphicheiral if it has symmetry $(-1, -1)$; and it is strongly $-$amphicheiral if it has symmetry $(J_-, -1)$.

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Similarly, we say a knot in a solid torus $(S^1 \times D^2, K)$ has symmetry $([\alpha, \epsilon_1], \epsilon_2)$ if there exists a self-homeomorphism of the solid torus of class α that maps the longitude class $[\lambda]$ to $\epsilon_1[\lambda]$ and restricts to a self-homeomorphism of K of the class ϵ_2.

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Lemma

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Lemma (Hartley)

Suppose P is a pattern and J is a non-trivial prime knot and neither J nor its mirror image is a companion of $P(U)$. Let $\alpha = \pm 1$ or $J_{\pm}$ and $\epsilon = \pm 1$. Then, $(S^3, P(J))$ has symmetry (α, ϵ) if and only if (S^3, J) has symmetry (α, ϵ_1) and $(S^1 \times D^2, P)$ has symmetry $([\alpha, \epsilon_1], \epsilon)$ for some $\epsilon_1 = \pm 1$.

Back to the satellite Miyazaki knot $K = P(J)$. Recall that K $-$amphicheiral means that (S^3, K) has symmetry $(-1, -1)$.

- Lemma of Hartley implies that (S^3, J) has symmetry $(-1, \epsilon_1)$ and $(S^1 \times D^2, P)$ has symmetry $([-1, \epsilon_1], -1)$ for some $\epsilon_1 = \pm 1$.
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Question: Are Miyazaki knots strongly $-$amphicheiral?

From the discussion in the previous slides, we see that all Miyazaki knots can be obtained from hyperbolic ones via iterated satellite operations. So we look for an inductive approach to the question.

Below is a partial result in this direction.

Proposition (Kim-W)

Suppose $K = P(J)$ is a Miyazaki knot with a strongly $-$amphicheiral companion J and a pattern of winding number 3. Then K is strongly $-$amphicheiral.

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Sketch of the proof

In light of Hartley's lemma, since J has symmetry $(J_-, -1)$, it suffices to prove that $(S^1 \times D^2, P)$ has symmetry $([J_-, -1], -1)$.

From the previous discussion, P is a fibered, unknotted pattern of winding number 3. By a result of [Magnus-Peluso, 1967], there are three such patterns up to isotopy in $S^1 \times D^2$, corresponding to the closure of conjugacy class of 3-braids

1. $\sigma_1\sigma_2$,
2. $\sigma_1^{-1}\sigma_2^{-1}$
3. $\sigma_1^{-1}\sigma_2$

The first two patterns of P give cable knots $K = J_{3,1}$ and $J_{3,-1}$, respectively, and one can show that K is not Miyazaki.

For the third case, we should observe directly that the closure of the braid $\sigma_1^{-1}\sigma_2$ has indeed the symmetry $([J_-, -1], -1)$. This finishes the proof.

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More general cases

What happens to a general satellite Miyazaki knot $K = P(J)$?

- The winding number $w = 2n + 1$ and $|w| \neq 1$.
- The fibered, unknotted pattern P is the closure of a $(2n+1)$-braid. (Hirasawa-Murasugi-Silver 2008)
- $P = \hat{\beta}$ is fibered for *homogeneous* braid β, i.e., each standard braid generator σ_i appears at least once in β and the exponent on σ_i has the same sign in each appearance in the braid word β. (Stallings 1978)

  E.g., $\prod_{i=0}^{2n} \sigma_{2n-i}^{(-1)^i}$ is a homogeneous $2n+1$-braid.

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An example

Example

Suppose P_n is the closure of a homogeneous $(2n+1)$-braid of the form $\prod_{i=0}^{2n} \sigma_{2n-i}^{(-1)^i}$. It has symmetry $([J_-, -1], -1)$.

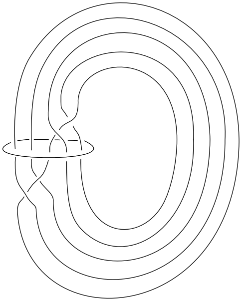

Figure: P_2 is the closure of a braid of the form $\sigma_4\sigma_3^{-1}\sigma_2\sigma_1^{-1}$. If we parameterize the meridional disk and the longitude of the solid torus by z and θ, respectively, then the map $f : (z, \theta) \to (-z, -\theta)$ is the desired orientation reversing involution.

Consequently, we construct an infinite family of satellite Miyazaki knots, all of which are strongly $-$amphicheiral.

Proposition (Kim-W)

Suppose J is the figure-eight knot, and P_n is the closure of a $(2n+1)$-braid of the form $\prod_{i=0}^{2n} \sigma_{2n-i}^{(-1)^i}$. Then the satellite knot $K = P_n(J)$ is Miyazaki and strongly $-$amphicheiral.

Question: To what extent could the above discussion be extended to spatial graphs?

Thank You

Thank you very much!