

International Workshop on Spatial Graphs 2016

Abstracts

– August 3 –

Milnor invariants of clover links

Kodai Wada (Waseda University)

J.P. Levine introduced a clover link to investigate the indeterminacy of the Milnor invariants of a link. It is shown that for a clover link, the Milnor numbers of length at most $2k + 1$ are well-defined if those of length at most k vanish, and that the Milnor numbers of length at least $2k + 2$ are not well-defined if those of length $k + 1$ survive. For a clover link c with the Milnor numbers of length at most k vanishing, we show that the Milnor number $\mu_c(I)$ for a sequence I is well-defined up to the greatest common divisor of $\mu_c(J)$'s, where J is a subsequence of I obtained by removing at least $k + 1$ indices. Moreover, if I is a non-repeated sequence with length $2k + 2$, the possible range of $\mu_c(I)$ is given explicitly. As an application, we give an edge-homotopy classification of 4-clover links.

On Milnor's $\bar{\mu}$ -invariants for handlebody-links

Yuka Kotorii (The University of Tokyo)

A handlebody-link is a disjoint union of handlebodies embedded in S^3 and HL-homotopy is an equivalence relation on handlebody-links generated by self-crossing changes. A. Mizusawa and R. Nikkuni classified the set of HL-homotopy classes of 2-component handlebody-links completely using the linking numbers for handlebody-links. In this talk, by using Milnor's $\bar{\mu}$ -invariants, we construct an invariant for handlebody-links and give a bijection between the set of HL-homotopy classes of n -component handlebody-links with some assumption and a quotient of the action of the general linear group on a tensor product of modules. This is joint work with Atsuhiko Mizusawa at Waseda University.

Parallel biquandle colorings for handlebody-knots and spatial graphs

Atsushi Ishii (University of Tsukuba)

A handlebody-knot is a handlebody embedded in the 3-sphere. Any handlebody-knot can be obtained as the regular neighborhood of a spatial trivalent graph. We introduce a notion of n -parallel biquandle and define colorings for handlebody-knots and spatial graphs. This is a joint work with Sam Nelson.

Triquasiles and Dual Graph diagrams for Oriented Spatial Graphs

Sam Nelson (Claremont McKenna College)

Dual graph diagrams are an alternative way of representing oriented knots, links and spatial graphs. In this talk we will use dual graph diagrams to define algebraic structures we call biquasiles and triquasiles and consider the integer-valued invariants of oriented spatial graphs they define.

Necklace diagrams and small cusps

Robert Haraway (University of Sydney)

In the first part of this talk, we review unpublished work of Berge on Heegaard diagram realizations of finite presentations of groups, based on earlier work of Whitehead and Zieschang.

In the second part of this talk, we discuss how this fits into ongoing work with Tom Crawford, Dave Gabai, Rob Meyerhoff, Nate Thurston, and Andrew Yarmola on enumerating hyperbolic 3-manifolds admitting a small maximal cusp.

Handlebody-knots and Moebius bands

Mario Eudave-Muñoz (Instituto de Matematicas, UNAM)

We give an explicit construction of an infinite family of genus 2 handlebody-knots in the 3-sphere, such that for each handlebody H in this family we have:

- 1) ∂H is incompressible in the exterior $E(H)$ of H . Recall that $E(H)$ is the 3-sphere minus the interior of H .
- 2) There is a properly embedded Moebius band B in $E(H)$, such that ∂B intersects all meridian disks of H .
- 3) The complement of the union of H and B is another handlebody.

It follows that the core of the Moebius band is an hyperbolic knot with a half-integral toroidal surgery. This gives an explicit construction of the family of knots $k(l, m, n, p)$ which have a non-integral toroidal surgery. I described this family of knots a long time ago, by using tangles and double branched covers.

In a recent paper, M. Ozawa and Y. Koda described all genus two handlebody-knots in the 3-sphere whose exterior is non-simple. One of the families described by them corresponds to the family we are constructing now. That is, we give an explicit description of one of the families described by Ozawa and Koda.

Heegaard Floer homology and spatial graphs

Yuanyuan Bao (The University of Tokyo)

We introduce recent developments of Heegaard Floer homology for spatial graphs, based on the independent work of Harvey and O'Donnol and the speaker.

Planar Legendrian Spatial Graphs

Danielle O'Donnol (Indiana University)

We will work in three-space with the standard contact structure. An embedded graph is Legendrian if it is everywhere tangent to the contact structure. I will give an overview of the invariants used in this area and talk about our recent work on several classification problems for planar Legendrian graphs. We have results on Legendrian simplicity, the Legendrian mirror problem, and stabilization equivalence. This is joint with Peter Lambert-Cole (Indiana).

Symmetries of graphs in homology spheres

Song Yu (Pomona College)

In this talk we will explore symmetries of spatial graphs in the family of homology spheres. We will generalize previous findings in the 3-sphere and see how concepts of rigidity, chirality and intrinsic linking interact with automorphisms of spatial graphs. Our results include a rigid symmetry theorem for 3-connected graphs, a classification of realizable automorphisms of complete graphs and examples of unrealizable automorphisms of Petersen graphs.

On Miyazaki’s fibered, negative amphicheiral knots

Zhongtao Wu (The Chinese University of Hong Kong)

In spatial graph theory, chirality is an interesting and important topic. In this talk, we discuss related questions for a special class of fibered, negative amphicheiral knots that were first studied by Miyazaki. In particular, we try to answer the open question of whether such knots are all strongly negative amphicheiral. This is a joint work with Min Hoon Kim.

Intrinsically asymmetric 2-colorings of spatial graphs

Erica Flapan (Pomona College)

In order to classify the symmetries of non-rigid molecules, we represent the molecules as spatial graphs Γ with different colored edges representing different types of molecular chains. We then analyze the homeomorphisms of (\mathbb{R}^3, Γ) which take edges of a given color to edges of the same color. With this motivation, we are interested in when the edges of a graph can be colored with one of two colors in a such a way that for any embedding Γ of the colored graph in \mathbb{R}^3 , no homeomorphism of (\mathbb{R}^3, Γ) induces a non-trivial color preserving automorphism on Γ . Such a 2-coloring is said to be intrinsically asymmetric. In addition to presenting our results on intrinsically asymmetric 2-colorings of graphs in \mathbb{R}^3 , we present results about intrinsically asymmetric 2-colorings of graphs in surfaces.

Arc index of spatial graphs

Sungjong No (Ewha Womans University)

Bae and Park found an upper bound on the arc index of prime links in terms of the minimal crossing number. We extend the definition of the arc presentation to spatial graphs and find an upper bound on the arc index $a(G)$ of any spatial graph G such that $a(G) \leq c(G) + e + b$, where $c(G)$ is the minimal crossing number of G , e is the number of edges, and b is the number of bouquet cut-components. This upper bound is lowest possible.

Stick number of spatial graphs

Minjung Lee (Korea university)

For a nontrivial knot K , Negami found an upper bound on the stick number $s(K)$ in terms of its crossing number $c(K)$ which is $s(K) \leq 2c(K)$. Later, Huh and Oh utilized the arc index $a(K)$ to determine a more precise upper bound $s(K) \leq 3/2c(K) + 3/2$. Furthermore, Kim, No and Oh found an upper bound on the equilateral stick number $s_=(K)$ as follows $s_=(K) \leq 2c(K) + 2$. As a sequel to this research program, we similarly define the stick number $s(G)$ and the equilateral stick number $s_=(G)$ of a spatial graph G , and present their upper bounds as follows: $s(G) \leq 3/2c(G) + 2e + 3b/2 - v/2$, $s_=(G) \leq 2c(G) + 2e + 2b - k$, where e and v are the number of edges and vertices of G , respectively, b is the number of bouquet cut-components, and k is the number of non-splittable components.

Enumeration Algorithm for Lattice Model

Seungsang Oh (Korea University)

In this talk we develop the state matrix recursion algorithm to enumerate various lattice models in deep and difficult problems of lattice combinatorics and statistics.

The restoring argument and the new intrinsically knotted graphs with 22 edges

Hyounghun Kim (Korea University)

A graph is intrinsically knotted if every embedding contains a knotted cycle. It is known that a intrinsically knotted graph is not 2-apex. The restoring argument is a constructing operation that constructs the graph from the given incomplete connection while avoiding being the graph 2-apex as much as possible. By using this operation, we classify triangle-free intrinsically knotted graphs with 22 edges which has exactly one degree 5 vertex.

Examples of Obstructions to Apex Graphs, Edge-Apex Graphs, and Contraction Apex Graphs

Mike Pierce (University California, Riverside)

A graph is apex if the removal of any vertex results in a planar graph. The Robertson-Seymour theorem affirms that there must be a finite number of minor-minimal non-apex (MMNA) graphs: graphs that are themselves not apex but whose every minor is apex. In this talk I will give an overview of our research into identifying and classifying this finite obstruction set. Specifically I will talk about classifying the MMNA graphs by their connectivity, about the results of a brute-force search for MMNA graphs, and about the effects of performing wye-triangle and triangle-wye transforms on non-apex graphs.

Additionally I will explore graphs with related properties like edge-apex (the deletion of an edge results in a planar graph), contraction-apex (the contraction of an edge results in a planar graph), and similar others, and how we can begin to identify the finite set of obstructions for these properties.

Combinatorics of intrinsic linking

Sergey Melikhov (Steklov Math Institute (Moscow))

We call a regular cell complex K *dichotomial* if to each cell C of K there corresponds another cell of K whose vertices are precisely all the vertices of K that are not in C . (The empty set is not counted as a cell.)

Then there exist precisely two 3-dimensional dichotomial complexes, whose 1-skeleta are the Kuratowski graphs K_5 and $K_{3,3}$ (i.e., the prohibited subgraphs/minors for planar graphs), and precisely six 4-dimensional dichotomial complexes, whose 1-skeleta are 6 out of the 7 graphs of the Petersen family (i.e. the prohibited minors for intrinsically linked graphs).

In general, all dichotomial complexes are homeomorphic to spheres. The n -skeleta of dichotomial $(2n + 1)$ -spheres are in some sense “minimal” among n -dimensional cell complexes non-embeddable in \mathbb{R}^{2n} , and the n -skeleta of dichotomial $(2n + 2)$ -spheres are in some sense “minimal” among n -dimensional cell complexes that are intrinsically linked in \mathbb{R}^{2n+1} . There are at least 13 dichotomial 5-spheres, whose 2-skeleta are pairwise distinct; only 3 of them are simplicial complexes.

I will review these old results of mine, focusing on low-dimensional examples, and discuss work in progress targeted at their improvement.

Minor theory for framed 4-valent graphs

Vassily Manturov (Bauman Moscow State Technical University)

Framed 4-graphs play a special role among all graphs (by “special” we mean that at each vertex two opposite half-edges are indicated). Such graphs appear as medial graphs for arbitrary graphs embedded in 2-surfaces, they also appear as singular links or knot projections; by using Euler tours they can be encoded by chord diagrams.

In 2004, the author proved Vassiliev’s conjecture on planarity of singular links saying that a framed 4-graph is non-planar if it contains a pair of cycles having exactly one transverse intersection (and possibly, no other intersection). Later, Nikonov proved that this theorem is equivalent to Pontrjagin-Kuratowski theorem, so, we have a first evidence of the following fact: classical problems for framed 4-graphs are easier as those for arbitrary graphs, and they are deeply connected.

Besides that, the author proved several criteria for non-planarity and non-embeddability into $\mathbb{R}P^2$ for different classes of framed 4-graphs. It is important that for framed 4-graphs one can consider “minor-closed properties”, where by “minor” we take a graph which can be obtained from the initial one by a finite sequence of smoothings at vertices (and deletions of separate components).

So, the notion of minor for framed 4-graphs is much simpler than that for arbitrary graphs. We will say some words about conjectural connections between the Robertson-Seymour-Thomas Theorem and its analog (still unproved) for framed 4-graphs.

As another evidence of the minor-closed property we shall take the planarity problem for framed 4-graphs admitting a source-sink structure; for such graphs, the non-planarity is equivalent to the existence of a non-trivial “sublink” for each embedding in \mathbb{R}^3 and to an existence of a non-trivial 2-component “sublink” with an odd linking number.

Free differential calculus and classical and twisted Alexander invariants of spatial graphs

Ryo Nikkuni (Tokyo Woman's Christian University)

For a finitely presented group of the deficiency one, it is well known that the first elementary ideal associated with the homomorphism from the group to the infinite cyclic group is principal and its generator is called the Alexander polynomial. In this talk, we give a brief review for free differential calculus on a finitely presented group of every positive deficiency and its applications to classical and twisted Alexander invariants of spatial graphs.