Prime knots whose arc index is smaller than the crossing number

Gyo Taek Jin

(joint with Hwa Jeong Lee)

Korea Advanced Institute of Science and Technology

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Arc presentation and arc index

An *arc presentation* of a knot or a link L is an ambient isotopic image of L contained in the union of finitely many half planes, called *pages*, with a common boundary line in such a way that each half plane contains a properly embedded single arc.

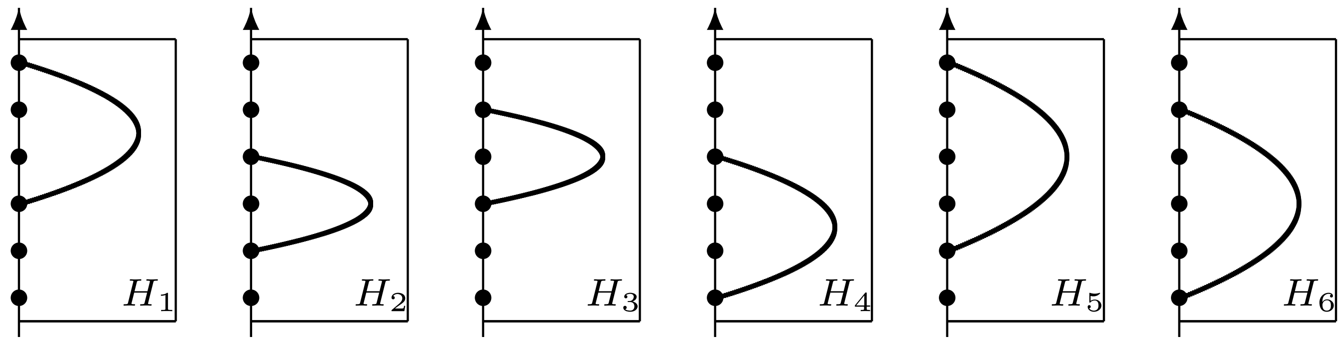


Figure 1: An arc presentation of the figure eight knot

The minimal number of pages among all arc presentations of a link L is called the *arc index* of L and is denoted by $\alpha(L)$.

Methods of describing arc presentation

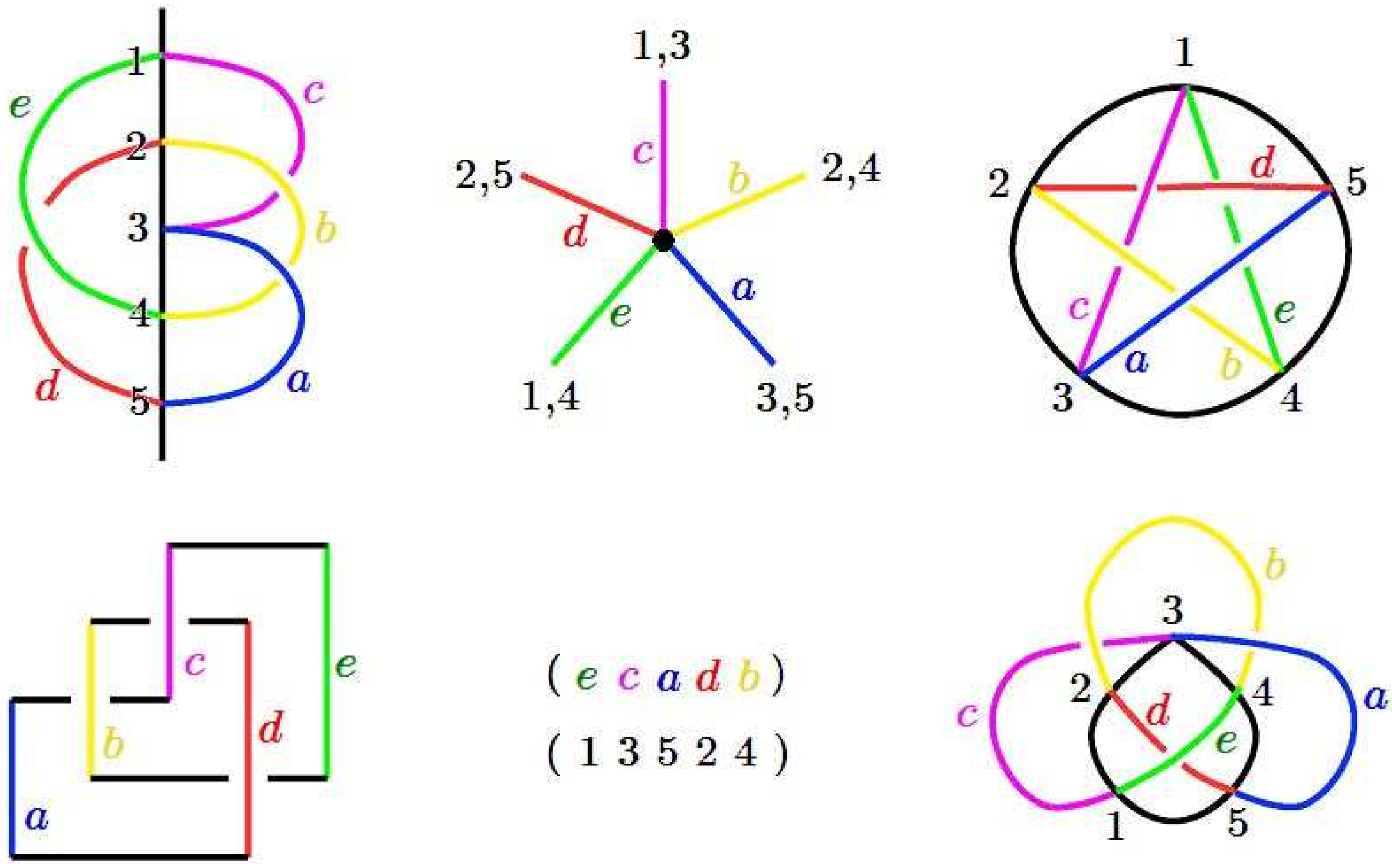


Figure 2: Representations of arc presentation

Brief History

- (Cromwell, 1995) Every link admits an arc presentation.
- (Nutt, 1999) All knots up to arc index 9 are identified.
- (Bae-Park, 2000)
  $\alpha(L) = c(L) + 2$ if and only if a non-split link L is *alternating*.
  (Knot-spoke diagrams are used for the proof.)
- (Beltrami, 2002) Arc index for prime knots up to 10 crossings are determined.
- (Jin et al., 2006) All prime knots up to arc index 10 are identified.
- (Ng, 2006) Arc index for prime knots up to 11 crossings are determined.
- (Jin-Park, 2007) All prime knots up to arc index 11 are identified.
  A prime link L is *nonalternating* if and only if $\alpha(L) \leq c(L)$.

Wheel Diagram

A *wheel diagram* is finite plane graph of straight edges which are incident to a single vertex. The projection of an arc presentation of a knot or a link into the xy-plane is of this shape.

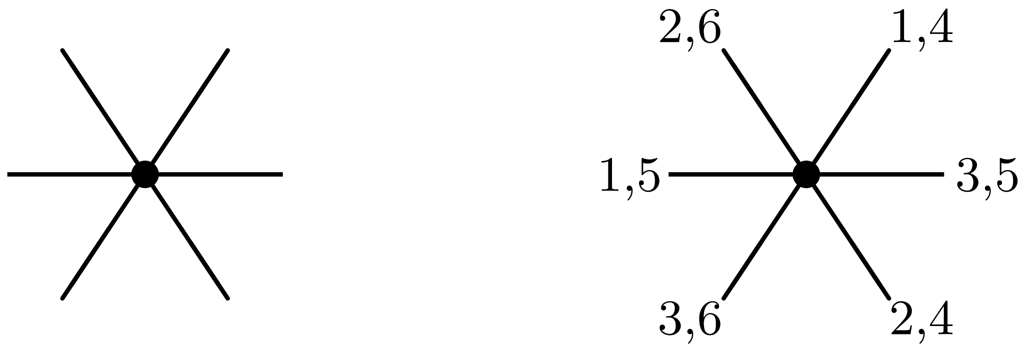


Figure 3: Wheel Diagrams of the figure-eight knot

For a wheel diagram with n edges to represent a knot or a link, each edge must be labeled with an unordered pair of distinct integers so that each of the integers, $1, 2, \cdots, n$ appear exactly twice in the wheel diagram. These number indicate the z-levels of the end point of the corresponding arcs.

Knot-spoke Diagram

A *knot-spoke diagram* $D^\star$ is a finite connected plane graph satisfying

1. There are three kinds of vertices in $D^\star$; a *distinguished vertex* v_0 with valency at least four, 4-valent vertices, and 1-valent vertices.
2. Every edge incident to a 1-valent vertex is also incident to v_0. Such an edge is called a *spoke*.

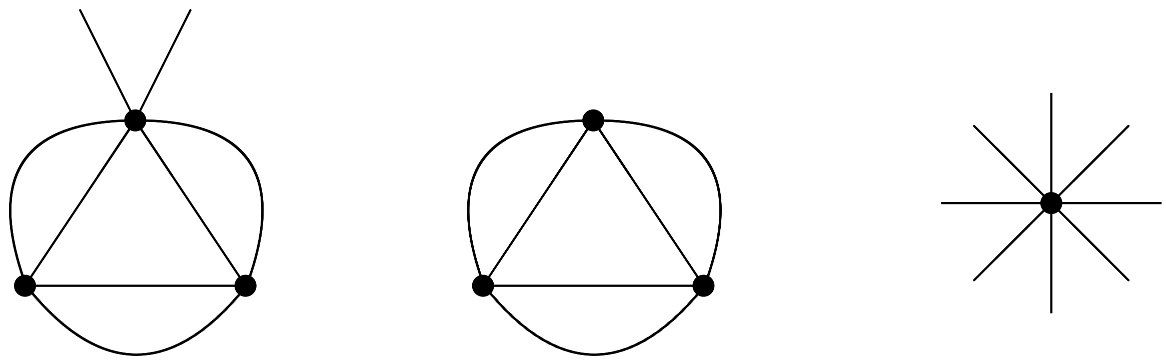


Figure 4: Knot-spoke diagrams

Prime knot-spoke diagrams

A knot-spoke diagram $D^\star$ is said to be *prime* if no simple closed curve meeting $D^\star$ in two interior points of edges separates multi-valent vertices into two parts.

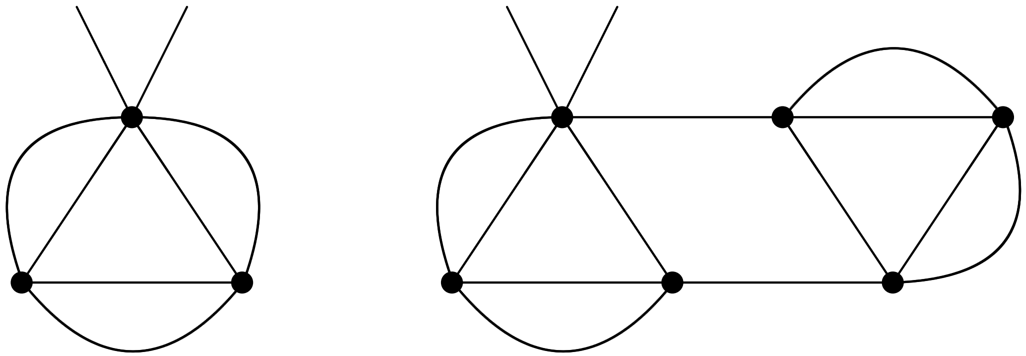


Figure 5: Prime diagram and non-prime diagram

Cut-point

A multi-valent vertex v of a knot-spoke diagram $D^\star$ is said to be a *cut-point* if there is a simple closed curve S meeting $D^\star$ in v and separating non-spoke edges into two parts.

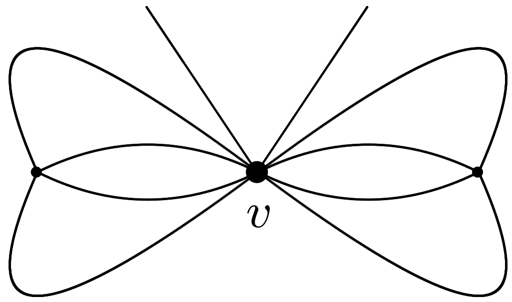


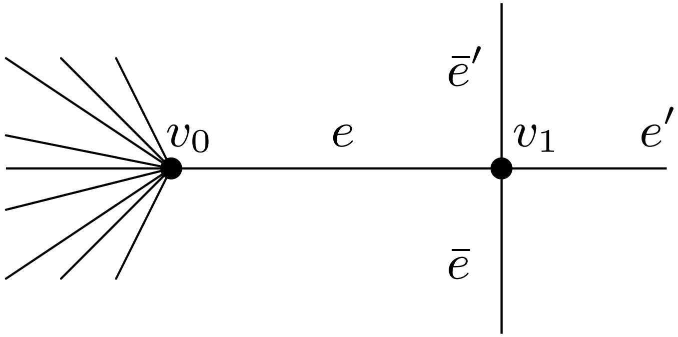
Figure 6: Cut-point

- A cut-point free knot-spoke diagram with more than one non-spoke edges cannot have a loop.
- If a prime knot-spoke diagram $D^\star$ has a cut-point, then the distinguished vertex v_0 must be the cut-point with valency bigger than four.

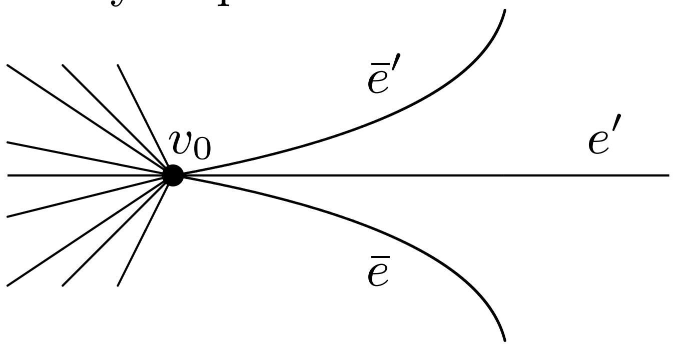
Contracting an edge incident to v_0

Let e be an edge of a cut-point free knot-spoke diagram $D^\star$ as in the figure. The knot-spoke diagram $(D^\star)_e$ is obtained by

- contracting e and
- replacing any simple loop thus created by a spoke.



Local diagram of $D^\star$ near e



Local diagram of $(D^\star)_e$ near v_0

Figure 7: Contraction of an edge in $D^\star$

A loop in a knot-spoke diagram is said to be *simple* if the other non-spoke edges are in one side of it.

$D^\star$ and $(D^\star)_e$

There are important facts to point out.

1. $D^\star$ and $(D^\star)_e$ represent the same knot or link.
2. The sum of the number of regions divided by the non-spoke edges and the number of spokes is unchanged.
3. $(D^\star)_e$ is prime if $D^\star$ is prime.

Wheel diagram with $c(D) + 2$ spoke

Starting from a knot diagram D, we end up with a knot-spoke diagram with $c(D)$ spokes and only one non-spoke edge which is a non-simple loop where $c(D)$ is the number of crossings in D.

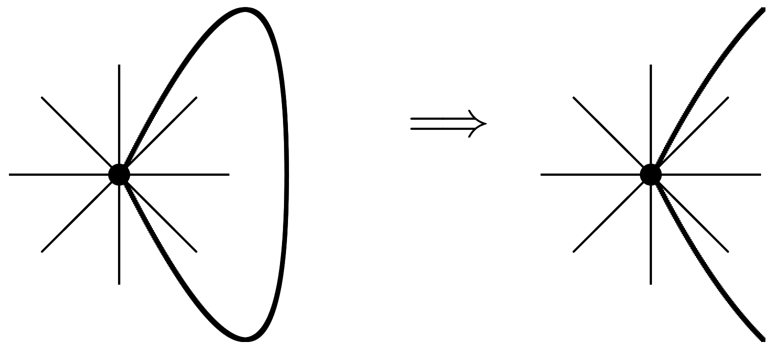
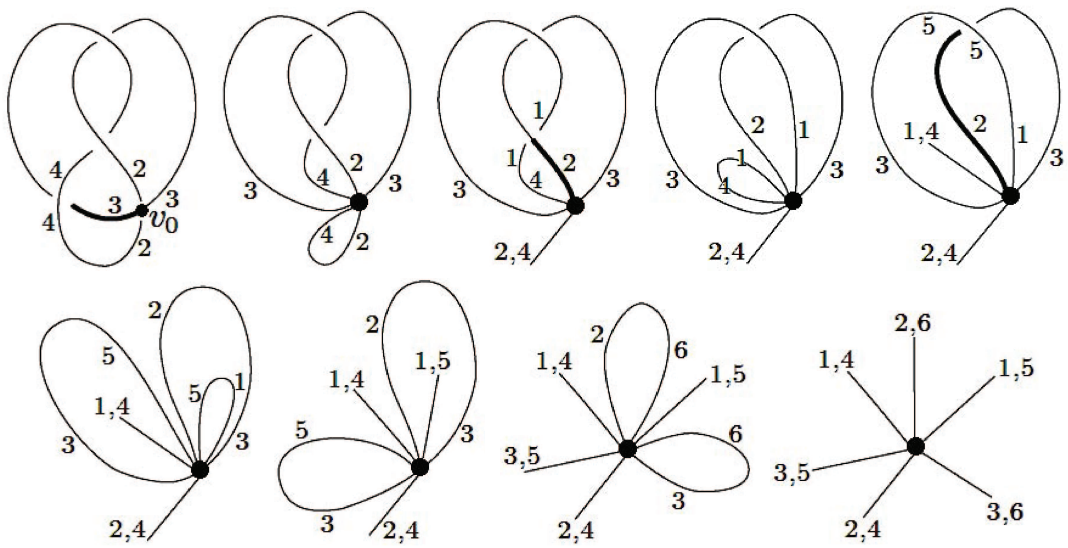


Figure 8: Folding the last non-spoke edge

The last non-spoke edge, which is a loop, is being folded to create two extra spokes. This shows the inequality $\alpha(L) \leq c(L) + 2$.

A process converting 4_1 into a wheel diagram

- Choose a vertex v_0 and put labels on the two edges meeting at v_0 , to assign vertical levels of the overpass and the underpass.
- Choose an edge e to contract and assign the label of a new level at the edges crossing e at the other end which is the lowest if the crossing is an undercrossing and the highest otherwise.
- Contract the edge and replace each simple loop with a spoke and label it with the two labels of the loop.



Filtered spanning Tree of a Knot Diagram

Let D be a knot diagram. We may consider D as a connected 4-valent plane graph with $c(D)$ vertices and $2c(D)$ edges.

A *spanning tree* of D is a tree which contains all the vertices of D.

A *filtered spanning tree* of D is an increasing sequence

$$T_0 \subset T_1 \subset T_2 \subset \cdots \subset T_{c(D)-1}$$

The *closure* of T_i, denoted by $\overline{T}_i$, is the subgraph of D obtained from T_i by adding the edges which are incident T_i at both ends.

Edges not contained in the spanning tree

An edge e of $\overline{T}_i \setminus \overline{T}_{i-1} \subset D$ is said to be *good* if e meets the edge $T_i \setminus T_{i-1}$ transversely at the vertex not contained in T_{i-1}.

An edge e of $\overline{T}_i \setminus \overline{T}_{i-1} \subset D$ is said to be *bad* if e meets the edge $T_i \setminus T_{i-1}$ vertically at the vertex not contained in T_{i-1}.

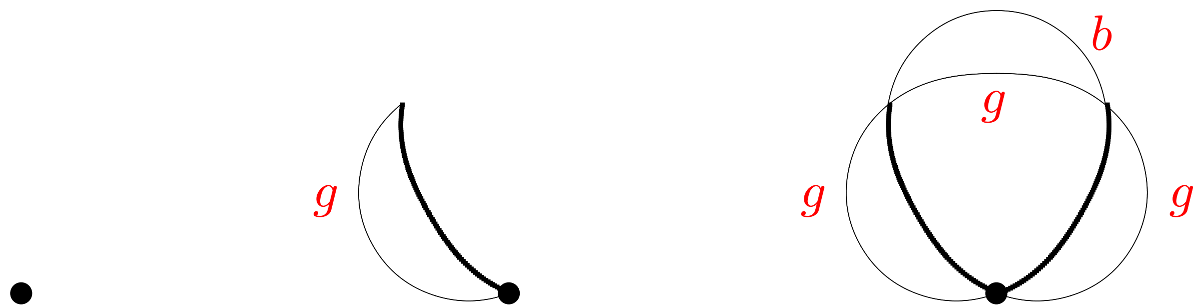


Figure 9: Good edges and a bad edge

Good filtered tree and Good filtered spanning tree

Let $T_0 \subset T_1 \subset \cdots \subset T_m$ be a filtered tree in a diagram D which does not span D. If the knot-spoke diagram obtained by contraction of the edges $e_i = T_i \setminus T_{i-1},\ i = 1, \ldots, m$ is cut-point free, we say that the *filtered tree is good*.

A *filtered spanning tree* $T_0 \subset T_1 \subset T_2 \subset \cdots \subset T_{c(D)-1}$ is said to be *good* if $T_0 \subset T_1 \subset \cdots \subset T_m$ is good filtered tree for each m, $1 \leq m \leq c(D) - 2$ and there is only one '*bad*' edge in D which belongs to $D \setminus \overline{T}_{c(D)-1}$.

Theorem 1 (Bae-Park, 2000) *A prime link diagram D admits a good filtered spanning tree and therefore we can obtain an arc presentation with $c(D) + 2$ arcs.*

Good filtered tree (Cont.)

Proposition 2 *Let $T_0 \subset T_1 \subset \cdots \subset T_m$ be a filtered tree in a diagram D which does not span D. Then the following are equivalent.*

1. *Every edge of $\overline{T}_m \setminus T_m$ is a good edge, and a sufficiently small neighborhood of $\overline{T}_m$ has connected exterior in D.*
2. *The filtered tree is good.*

Corollary 3 *Let $T_0 \subset T_1 \subset \cdots \subset T_m$ be a good filtered tree in a diagram D which does not span D. Let e be an edge in D such that $T_m \cap e$ is a single vertex, so that $T_m \cup e$ is a tree. If $T_0 \subset T_1 \subset \cdots \subset T_m \subset (T_m \cup e)$ is not a good filtered tree, then one of the following holds.*

- *$\overline{T_m \cup e}$ has a bad edge.*
- *A sufficiently small neighborhood of $\overline{T_m \cup e}$ has disconnected exterior in D.*

Cutting arc

Let T be a filtered tree in D which does not span D. A simple arc Γ is called a *cutting arc* of T if it is satisfies the following conditions.

1. $\Gamma \cap D$ consists of the endpoints of Γ which are two distinct vertices of T.
2. A proper subcollection of edges of $D \setminus \overline{T}$ is enclosed by the simple closed curve $\overline{\Gamma}$ constructed by Γ and the path in T joining the endpoints of Γ.

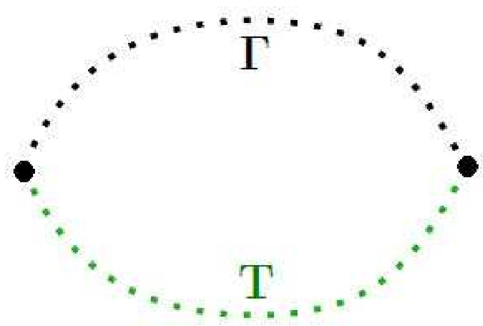


Figure 10: Cutting arc of a filtered tree

Doubly good edges

A good edge $e \subset \overline{T}_i \setminus \overline{T}_{i-1}$ is said to be *doubly good* if the three edges e, $e_i = T_i \setminus T_{i-1}$, and $e_{i-1} = T_{i-1} \setminus T_{i-2}$ together bound a nonalternating triangular region in D/T_{i-2}.

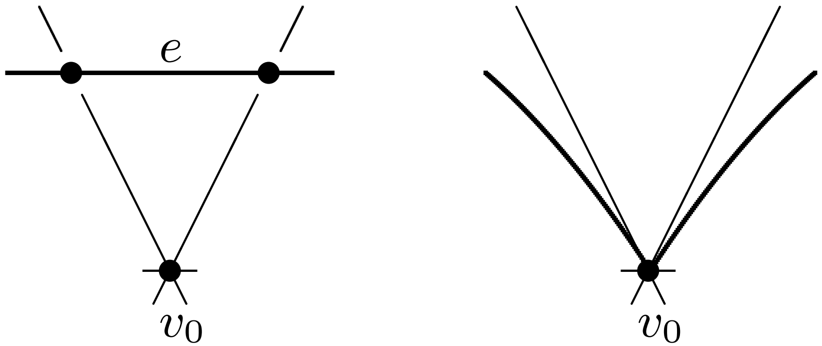


Figure 11: Doubly good edge on a nonalternating triangular region

The doubly good edge on the filtered tree corresponds to removable spoke in the knot-spoke diagram obtained by contraction of edges in T_i.

Doubly good edges (Cont.)

It is known that every prime nonalternating diagram admits a good filtered spanning tree having at least two doubly good edges.

Theorem 4 (Jin-Park, 2007) *A prime link L is nonalternating if and only if $\alpha(L) \leq c(L)$.*

Theorem 5 *A prime diagram D of a nonalternating knot has a good filtered spanning tree which has at least two doubly good edges. Furthermore, if there are d doubly good edges, then one can obtain an arc presentation with $c(D) + 2 - d$ arcs.*

Goal

Arc presentation	Wheel diagram	**Good filtered spanning tree**
properly simple arc	spoke	good edge
removable arc	removable spoke	**doubly good edge**

Goal : **To construct a good filtered tree to find as many doubly good edges as possible.**

Supporting arc and String $\overrightarrow{ve}$

For two regions R and S in a diagram, an arc Δ is said to be a *supporting arc* of R and S if Δ consists of at least 3 edges and one of the end edges of Δ is one of the boundary edges of R and the other is one of the boundary edges of S.

A string from a vertex v extending e in a knot diagram D is a portion of D that goes from v passing through e along D and is denoted by $\overrightarrow{ve}$.

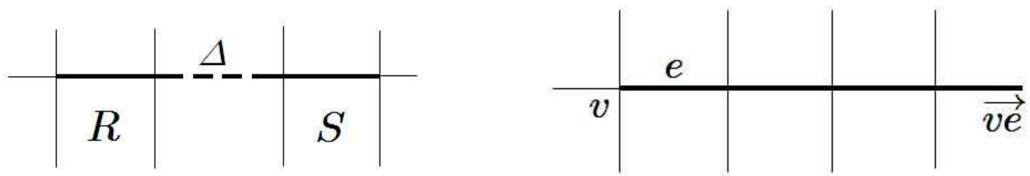
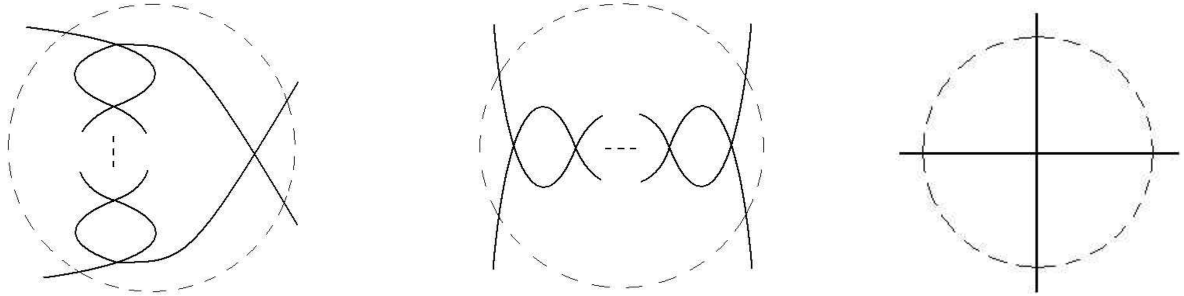


Figure 12: Supporting arc and String $\overrightarrow{ve}$

Three types of nonalternating diagram

Let $n \geq 2$. A nonalternating knot diagram D is said to be *$(n, 1)$-nonalternating* if it can be decomposed of two alternating tangles one of which is an $(n, 1)$-tangle.
Let $n \geq 1$. A nonalternating knot diagram D is said to be *n-nonalternating* if it can be decomposed of two alternating tangles one of which is an n-tangle.
A 1-nonalternating diagram is also called an *almost alternating diagram*.



(a) The $(n, 1)$-tangle (b) The n-tangle (c) The 1-tangle

Figure 13: Tangles

Theorem A

Let D be a prime $(n, 1)$-nonalternating minimal crossing knot diagram having a nonalternating triangular region with some edges and regions labeled as in Figure 14 for some integer $n \geq 2$. Then $\alpha(D) < c(D)$ if D satisfies the two conditions below:

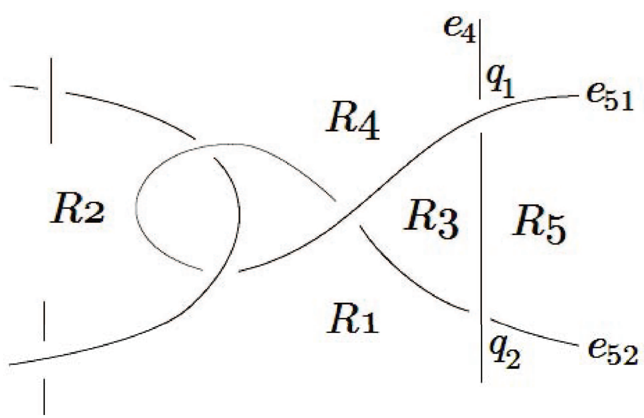


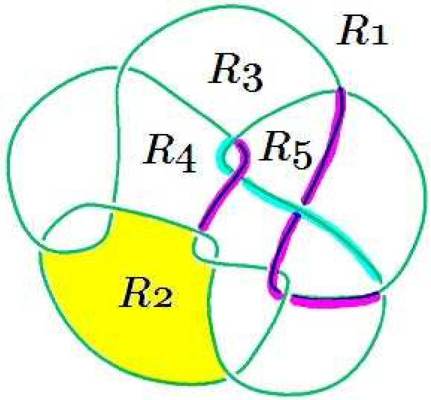
Figure 14: $(2, 1)$-nonalternating diagram

Theorem A (Cont.)

1. The string $\overrightarrow{q_1 e_4}$ and at least one of the two strings $\overrightarrow{q_1 e_{51}}$, $\overrightarrow{q_2 e_{52}}$ meet at a crossing before they become incident to the region R_2, or there is a supporting arc of R_4 and R_5 which does not contain any edge of ∂R_2 and ∂R_3.

2. At least one of the three strings $\overrightarrow{q_1 e_4}, \overrightarrow{q_1 e_{51}}, \overrightarrow{q_2 e_{52}}$ is incident to R_2 before or at the same time to R_1, or there is a supporting arc of R_4 and R_5, not incident to R_3, whose extension is incident to R_2 before or at the same time to R_1.

Diagrams on which Theorem A can be applied



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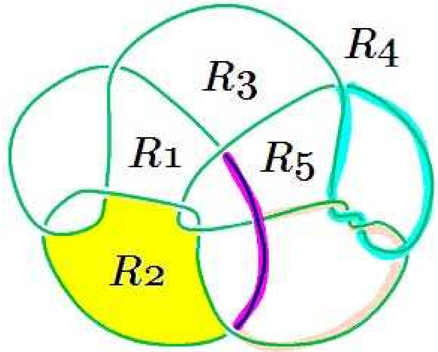
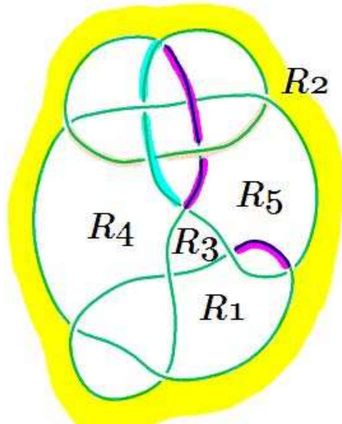
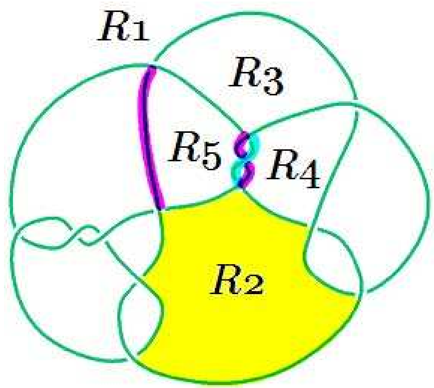


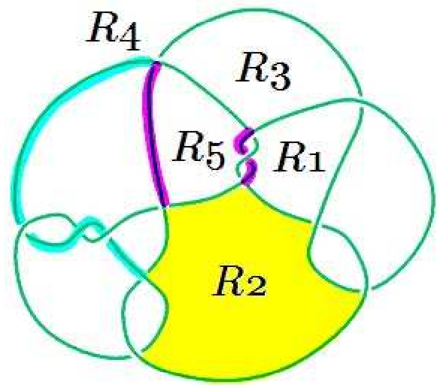
13n1404



13n1042



13n1974



13n1974

Theorem B

Let D be a prime, (n)-nonalternating and minimal crossing knot diagram having a nonalternating triangular region. If D satisfies the condition $1, 2,$ and 3 where the regions and edges near the nonalternating triangular region are labeled as in Figure 15. Then $\alpha(D) < c(D)$.

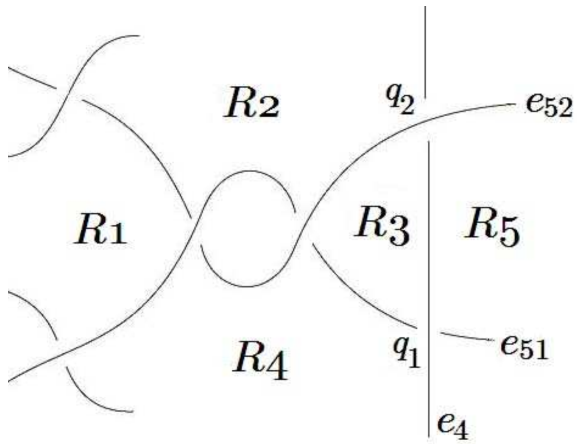


Figure 15: 2-nonalternating diagram

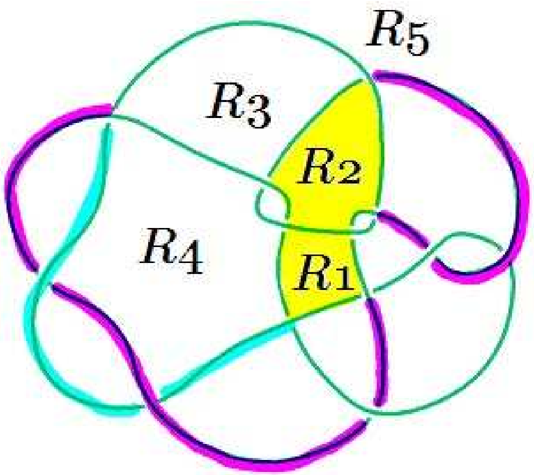
Theorem B (Cont.)

1. String $\overrightarrow{q_1 e_4}$ and at least one of the two strings $\overrightarrow{q_1 e_{51}}$, $\overrightarrow{q_2 e_{52}}$ meet at a crossing before they become incident to the regions R_1 or R_2, or there is a supporting arc of R_4 and R_5 which does not contain any edge of R_1, R_2 and R_3.

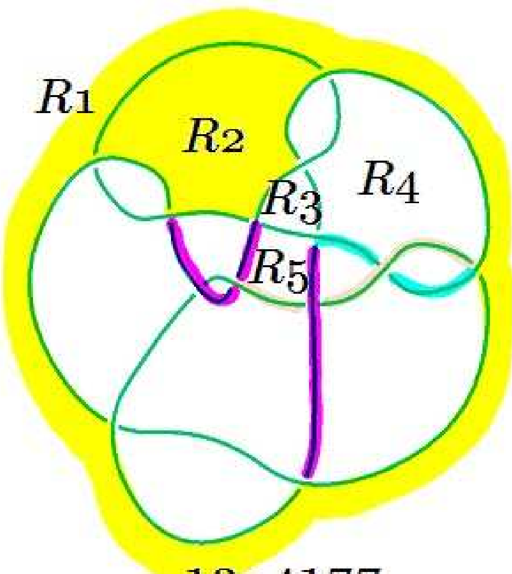
2. At least one of the three strings $\overrightarrow{q_1 e_4}$, $\overrightarrow{q_1 e_{51}}$, $\overrightarrow{q_2 e_{52}}$ is incident to R_2 before or at the same time to R_1, or there is a supporting arc of R_4 and R_5, not incident to R_3, whose extension is incident to R_2 before or at the same time to R_1.

3. R_2 is bounded by at least $n + 3$ edges.

Diagrams on which Theorem B can be applied



13n4276



13n4177

Theorem C

Let D be a prime, almost alternating, and minimal crossing knot diagram having a nonalternating triangular region. If D satisfies the condition $1, 2, 3$ where the regions and edges near the nonalternating triangular region are labeled as in Figure 16. Then $\alpha(D) < c(D)$

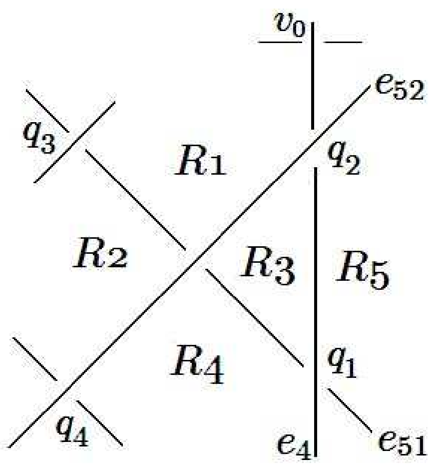
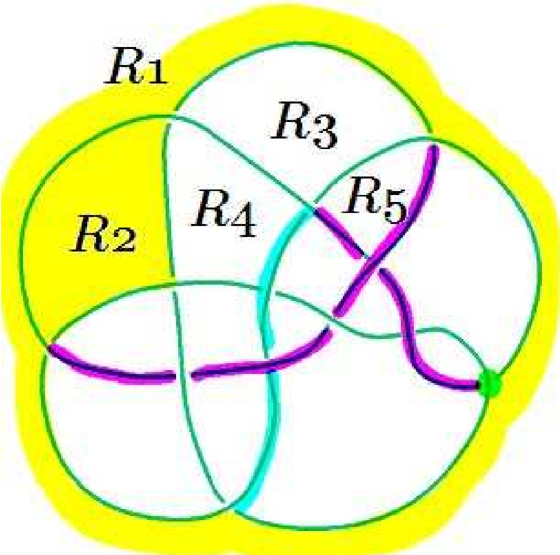


Figure 16: Almost alternating diagram

Theorem C (Cont.)

1. $\overrightarrow{q_1e_4}$ and at least one of $\overrightarrow{q_1e_{51}}$, $\overrightarrow{q_2e_{52}}$ meet at a crossing before they become incident to the regions R_1 or R_2, or there is a supporting arc of R_4 and R_5 which does not contain any edge of R_1, R_2 and R_3.
2. At least one of the three strings $\overrightarrow{q_1e_4}$, $\overrightarrow{q_1e_{51}}$, $\overrightarrow{q_2e_{52}}$ is incident to R_2 before or at the same time to R_1, or there is a supporting arc of R_4 and R_5, not incident to R_3, whose extension is incident to R_2 before or at the same time to R_1.
3. At least one of $\overrightarrow{q_1e_4}$, $\overrightarrow{q_1e_{51}}$ is incident to R_1 at v_0 for the first time without being incident to R_2.

A diagram on which Theorem C can be applied



13n4031

A sketch of proof of Theorem A

Let D' be the diagram obtained from D by a type 3 Reidemeister move over the region R_3 as in Figure 17.

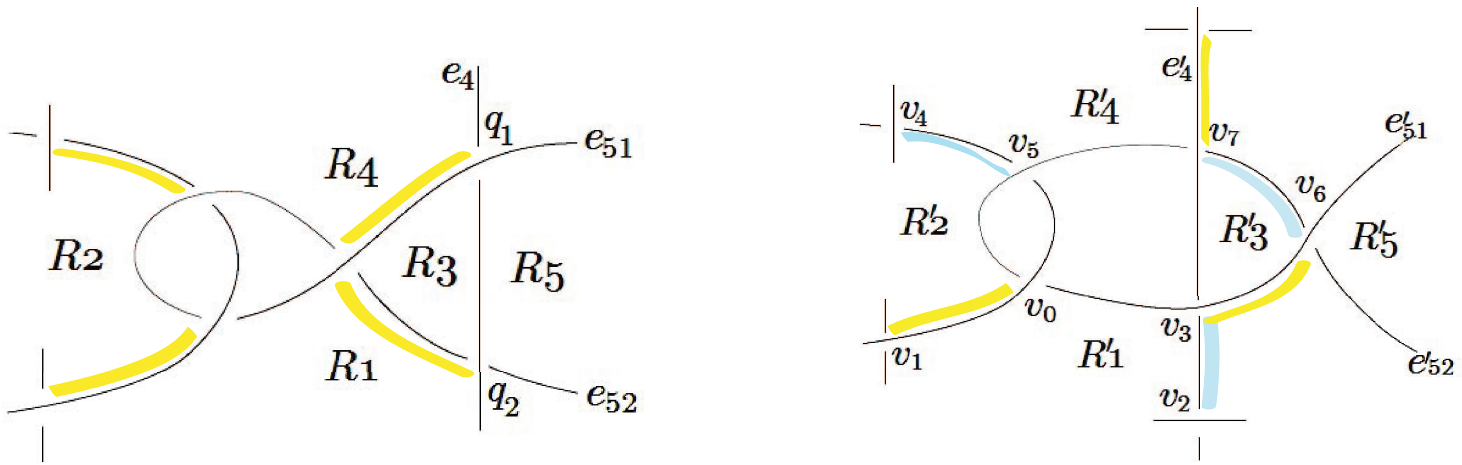


Figure 17: $(2, 1)$-nonalternating diagram after a type 3 Reidemeister move

We construct a **good filtered tree whose closures gradually contain $\partial R'_1, \partial R'_2$ and $\partial R'_3$**. Let $\overline{v_i v_j}$ denote the edge joining v_i and v_j. The edges **$\overline{v_2 v_3}$, $\overline{v_4 v_5}$ and $\overline{v_6 v_7}$ will become doubly good edges**.

A sketch of proof of Theorem B

Let D' be the diagram obtained from D by a type 3 Reidemeister move over the region R_3 as in Figure 18.

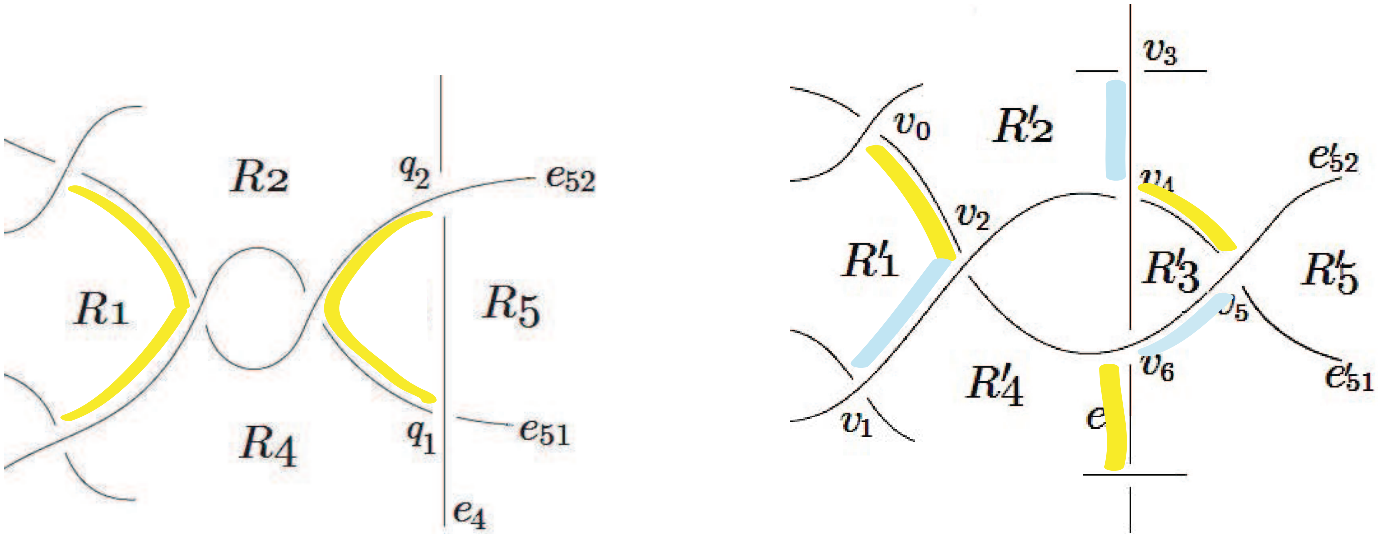


Figure 18: 2-nonalternating diagram after a type 3 Reidemeister move

We construct a **good filtered tree whose closures gradually contain** $\partial R'_1, \partial R'_2$ **and** $\partial R'_3$. The edges $\overline{v_1 v_2}$, $\overline{v_3 v_4}$ **and** $\overline{v_5 v_6}$ **will become doubly good edges**.

A sketch of proof of Theorem C

Let D' be the diagram obtained from D by a type 3 Reidemeister move over the region R_3 as in Figure 19.

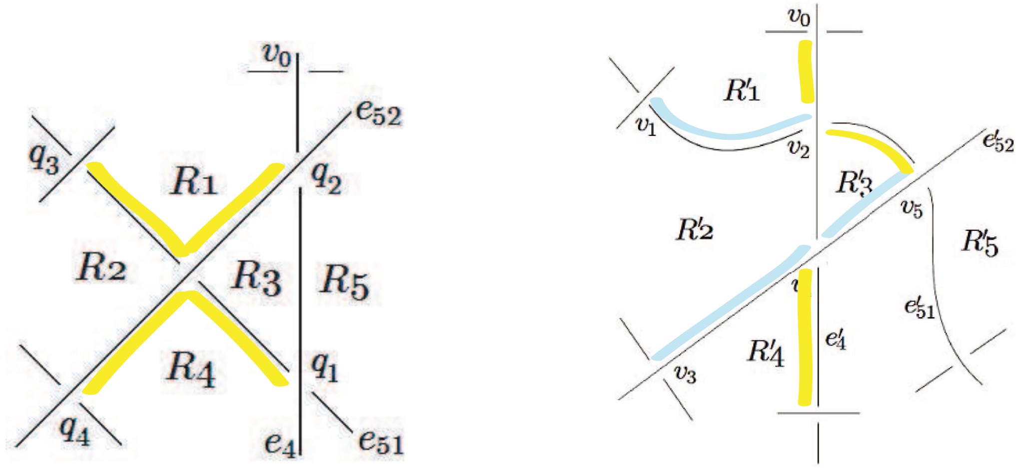


Figure 19: Almost alternating diagram after a type 3 Reidemeister move

We construct a **good filtered tree whose closures gradually contain $\partial R'_1, \partial R'_2$ and $\partial R'_3$**. The edges **$\overline{v_1v_2}$, $\overline{v_3v_4}$ and $\overline{v_4v_5}$ will become doubly good edges**.

An Example of Theorem A

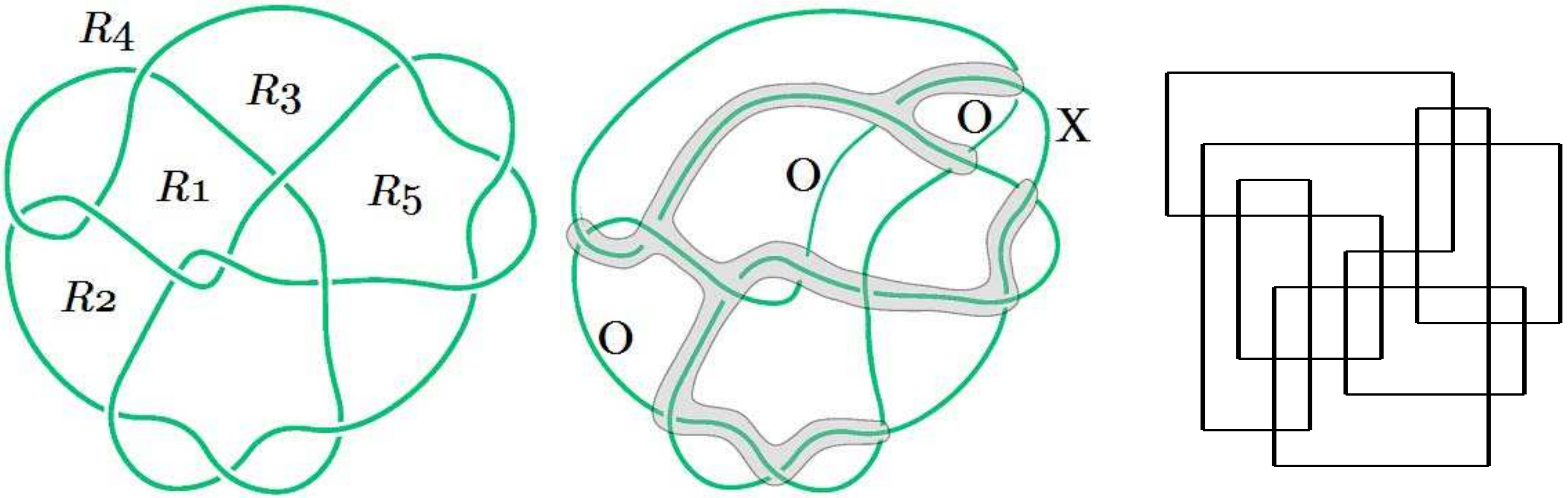
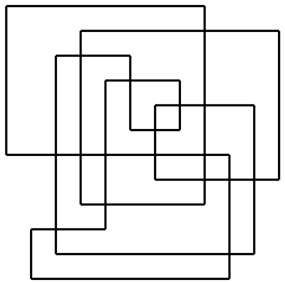
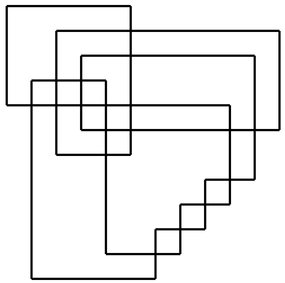


Figure 20: (2, 1)-nonalternating diagram : $13n2004$

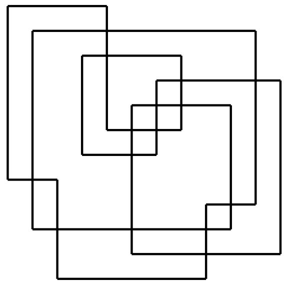
Examples of Theorem A (knots with arc index 12)



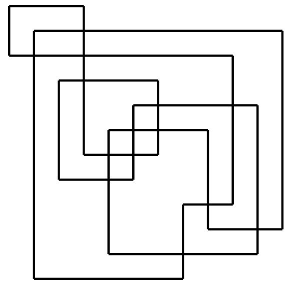
$13n563$



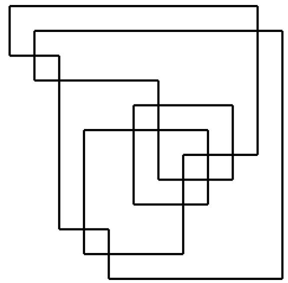
$13n572$



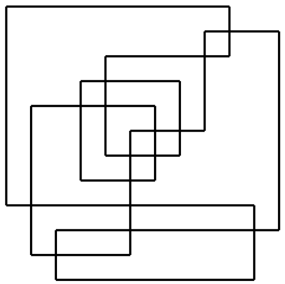
$13n651$



$13n652$



$13n689$



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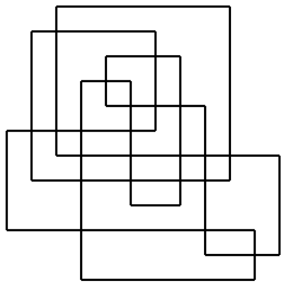

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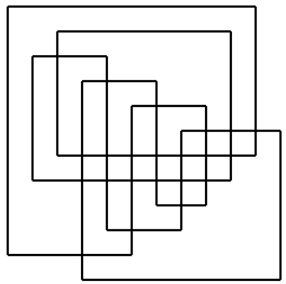

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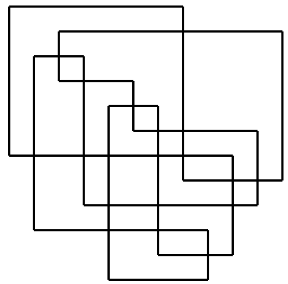

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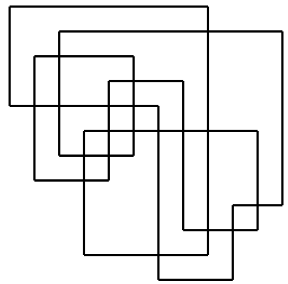

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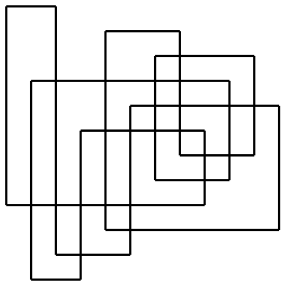

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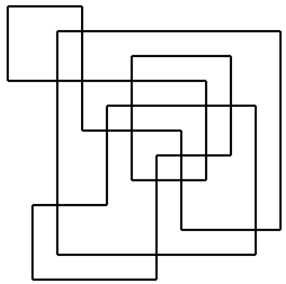

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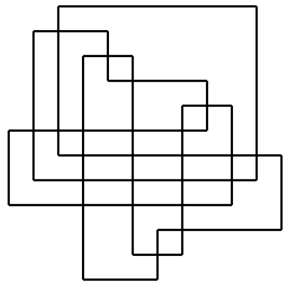

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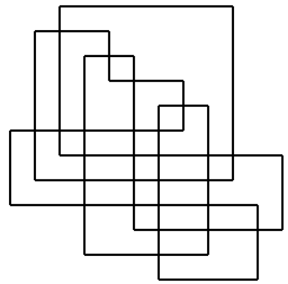

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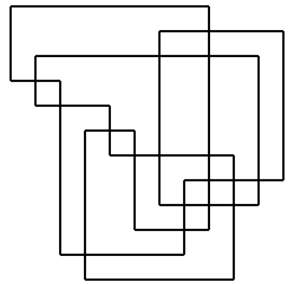

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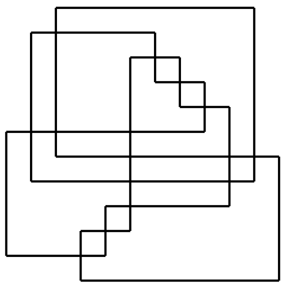

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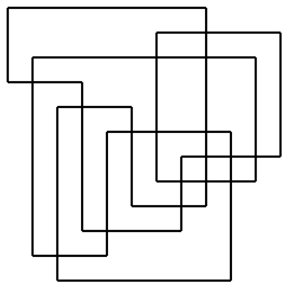

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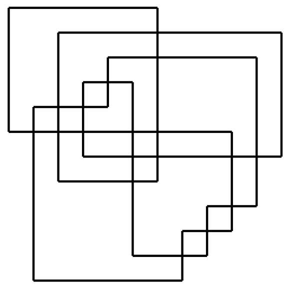

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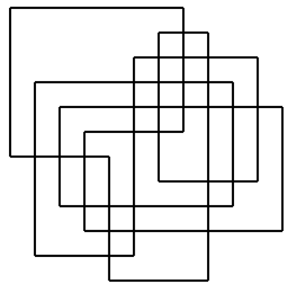

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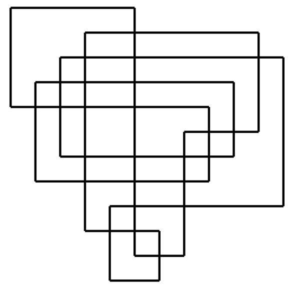

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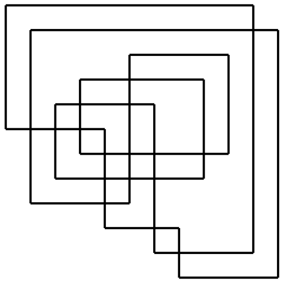

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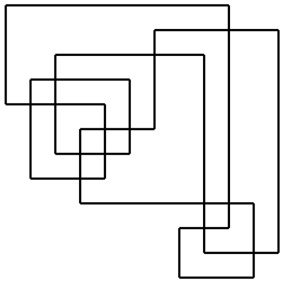

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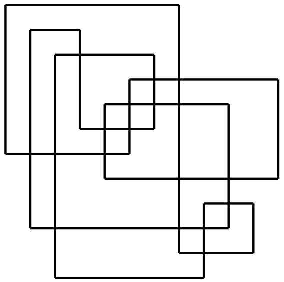

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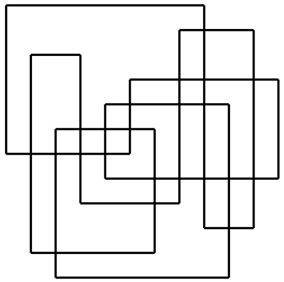

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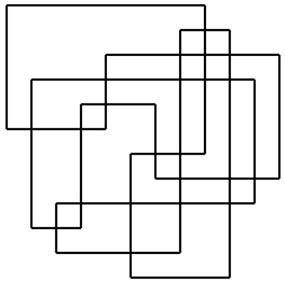

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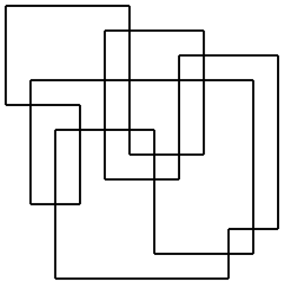

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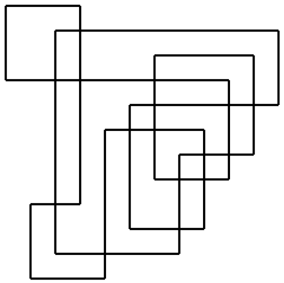

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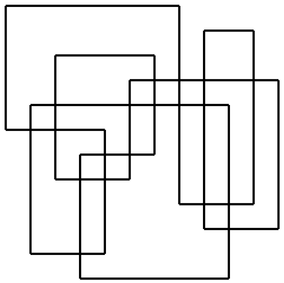

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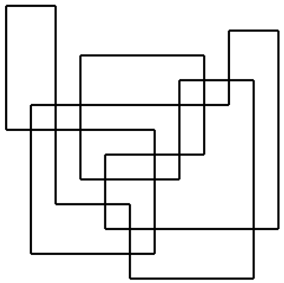

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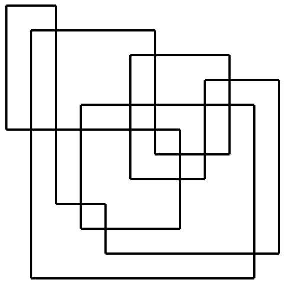

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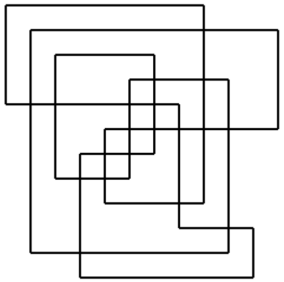
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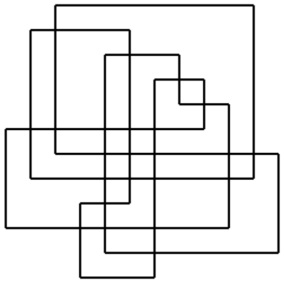
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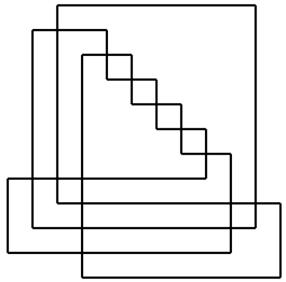
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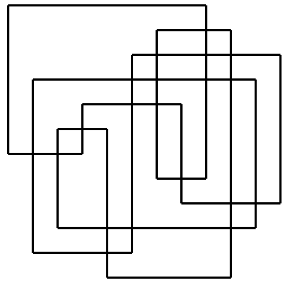
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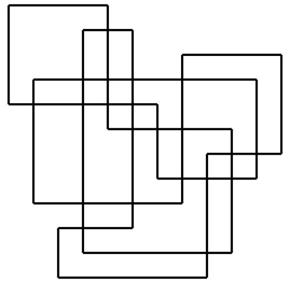
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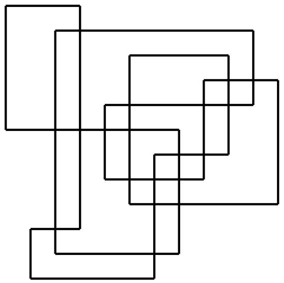
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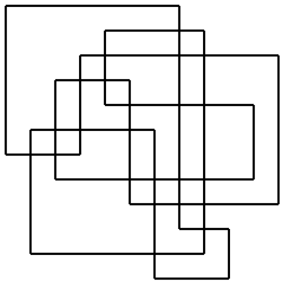
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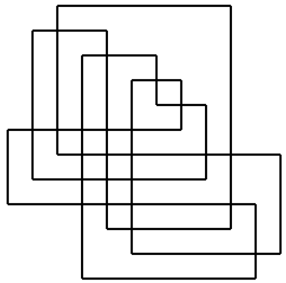
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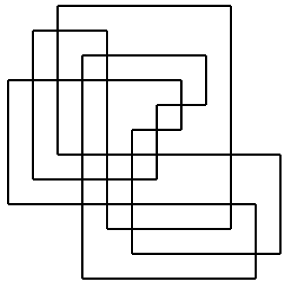
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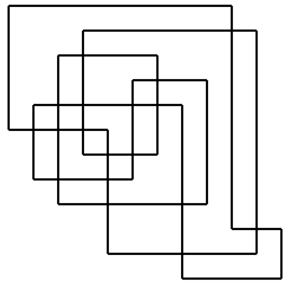
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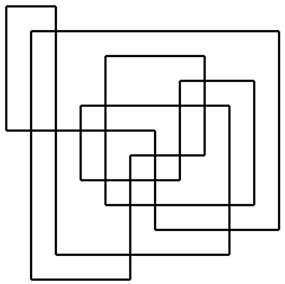
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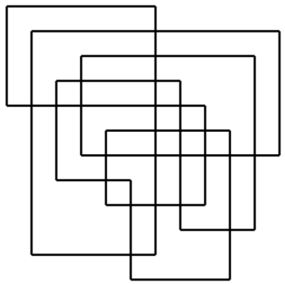
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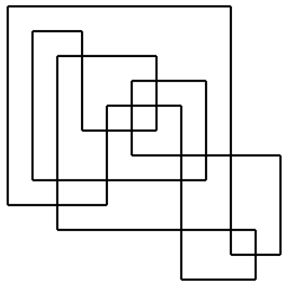
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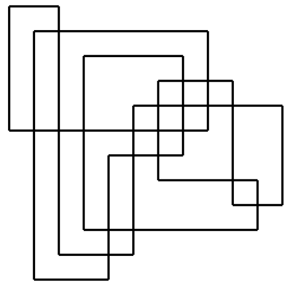
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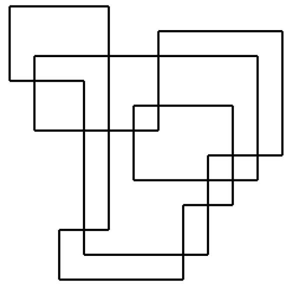
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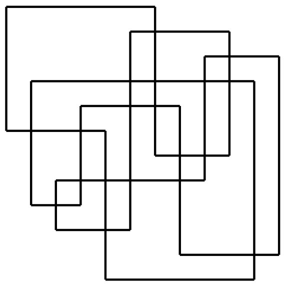
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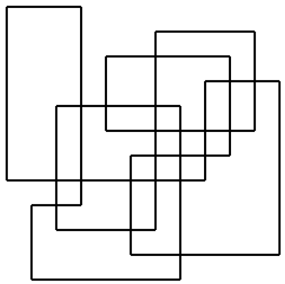
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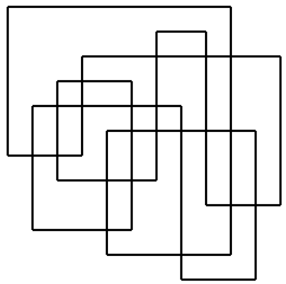
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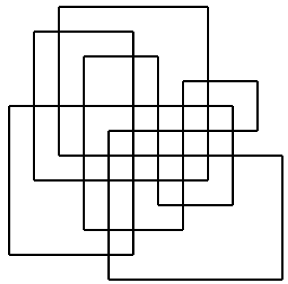
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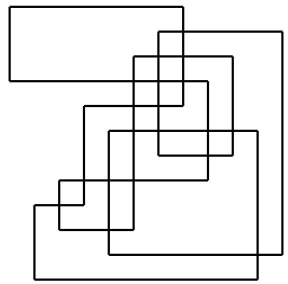
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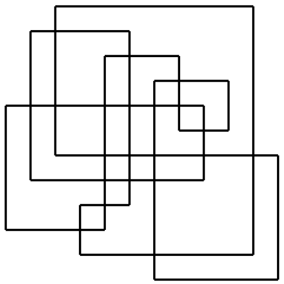

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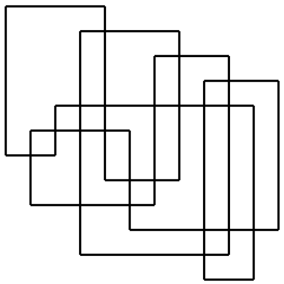

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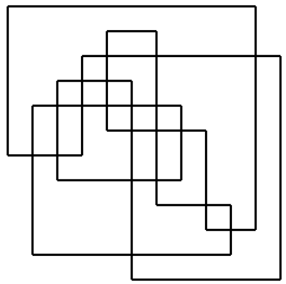

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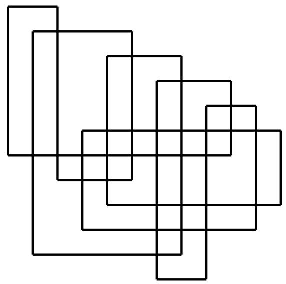

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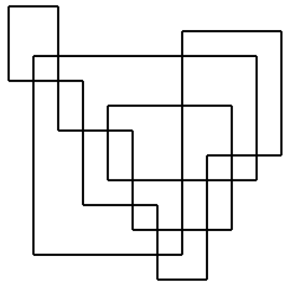

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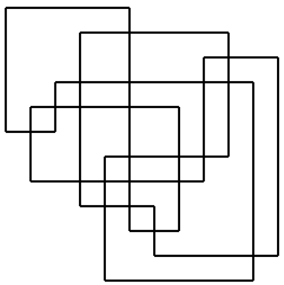

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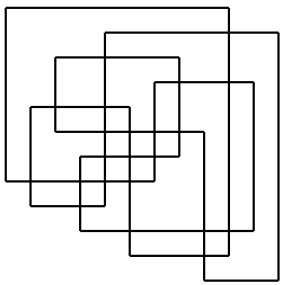

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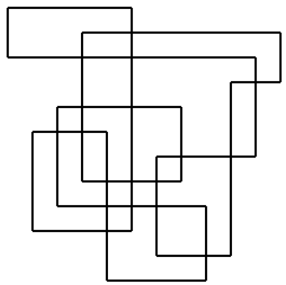

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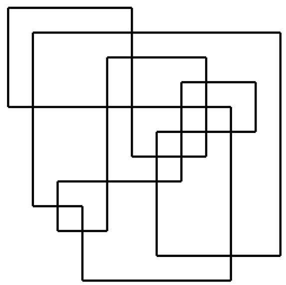

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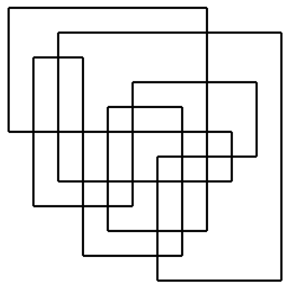

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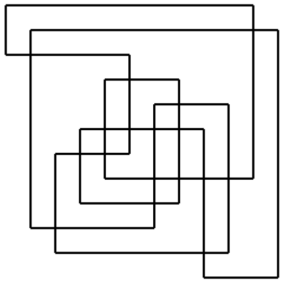
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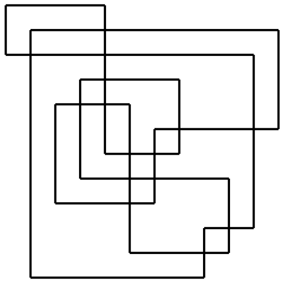
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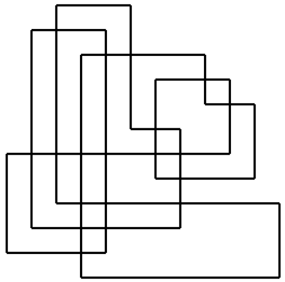
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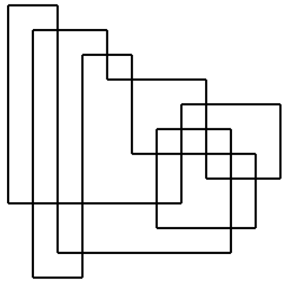
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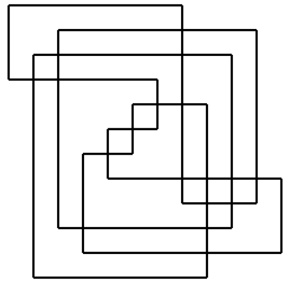
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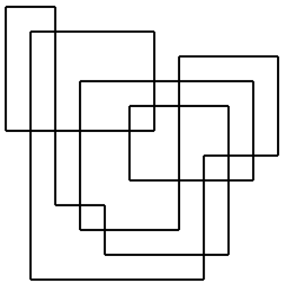
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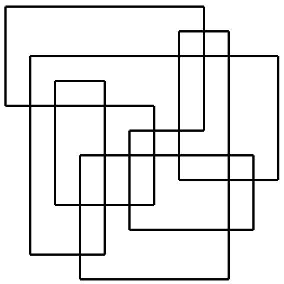
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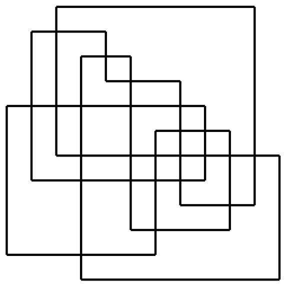
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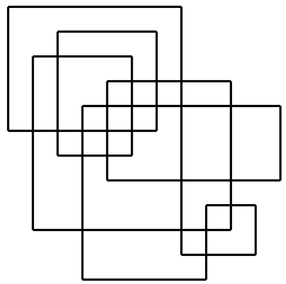
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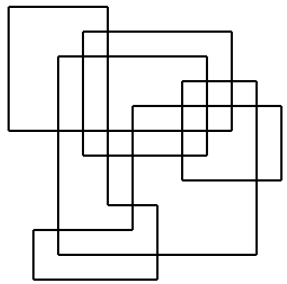
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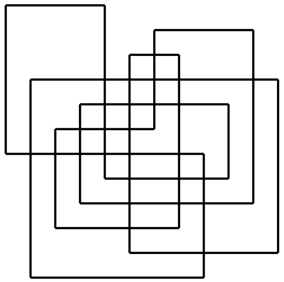

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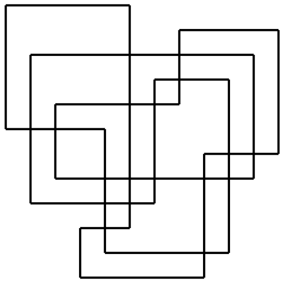

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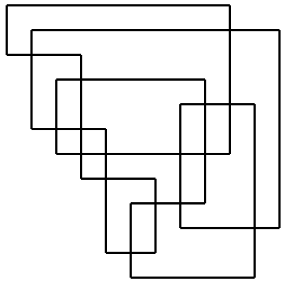

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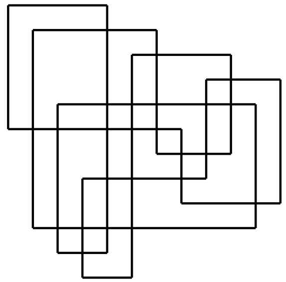

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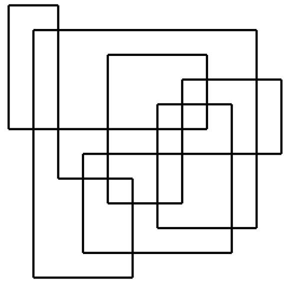

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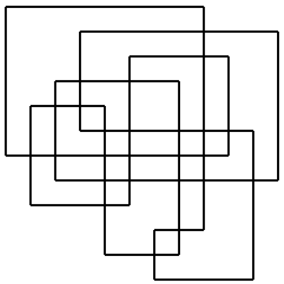

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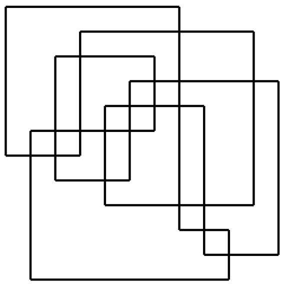

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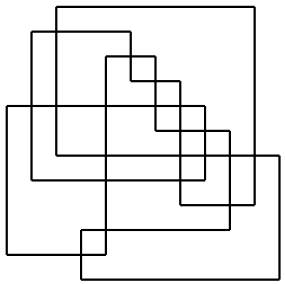

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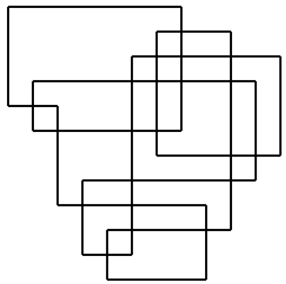

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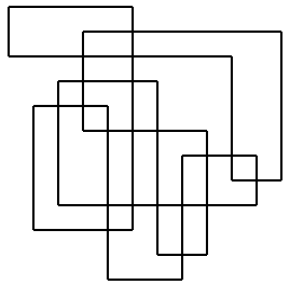

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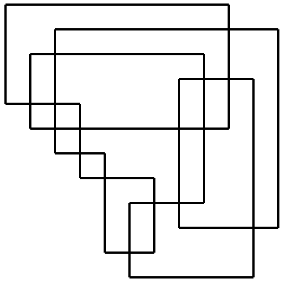

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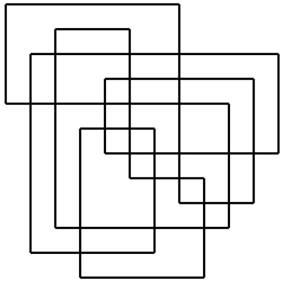

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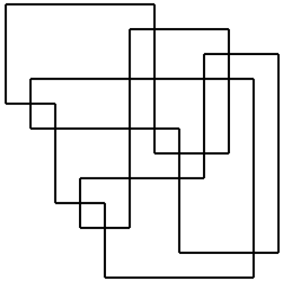

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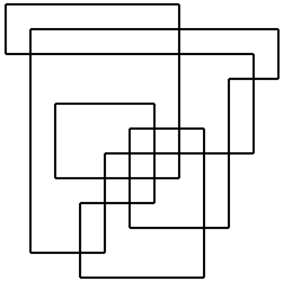

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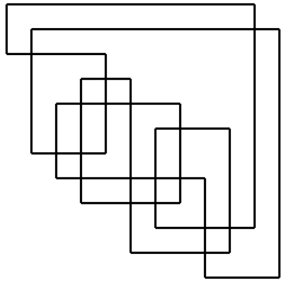

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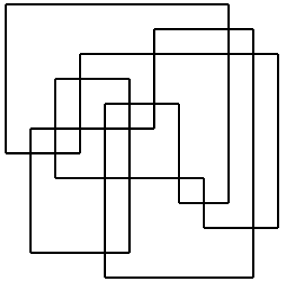

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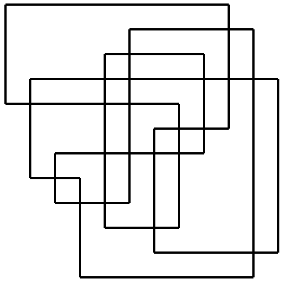

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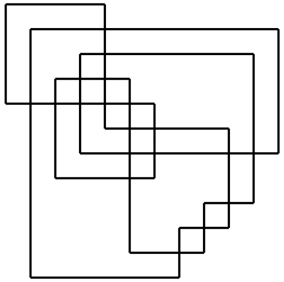

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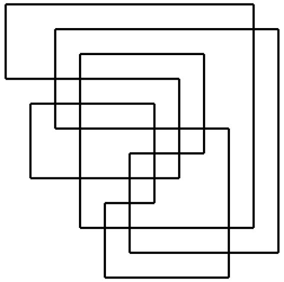

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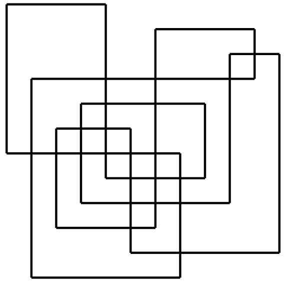

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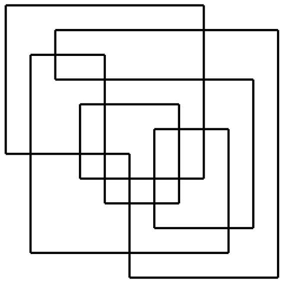

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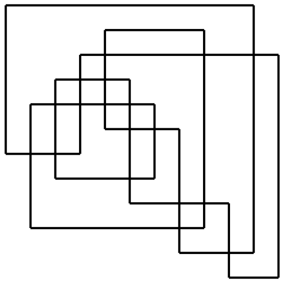

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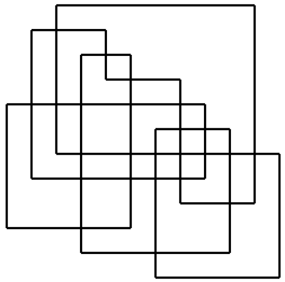

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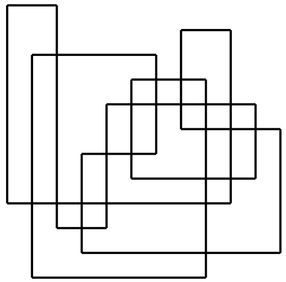

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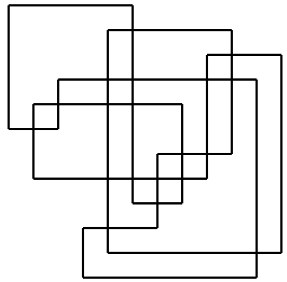

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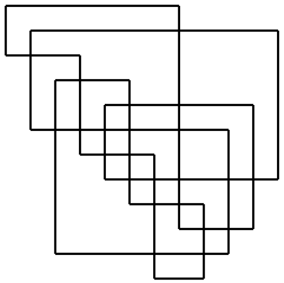

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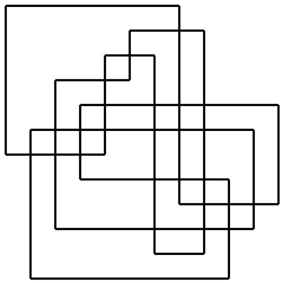

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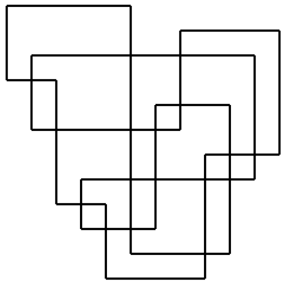

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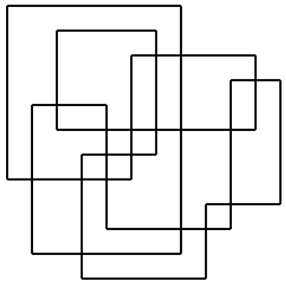

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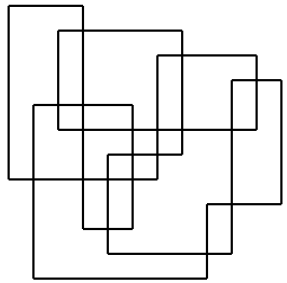

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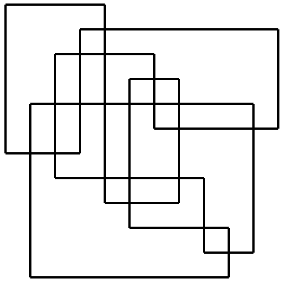

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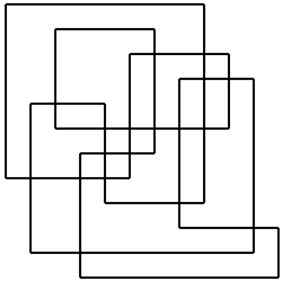

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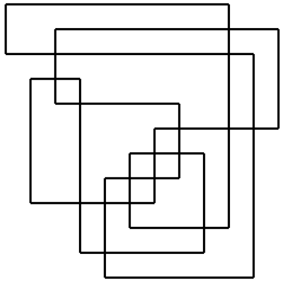

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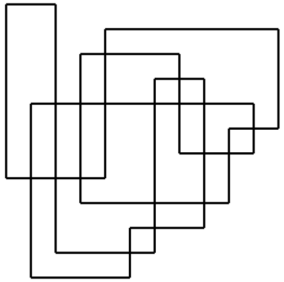

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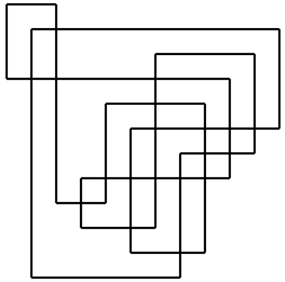

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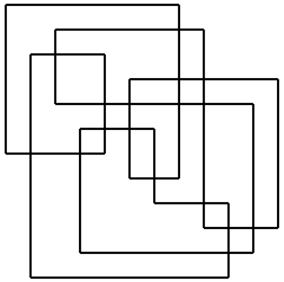

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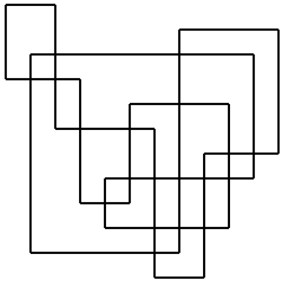

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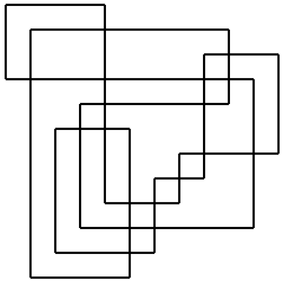

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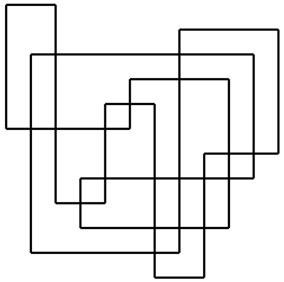

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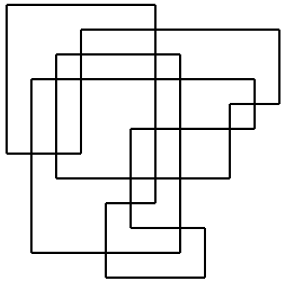

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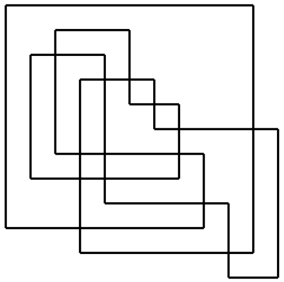

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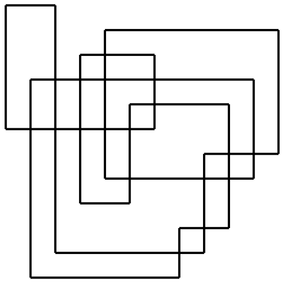

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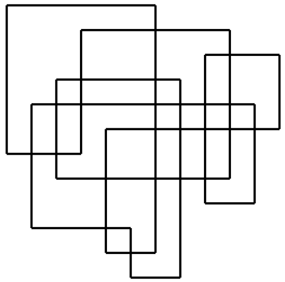

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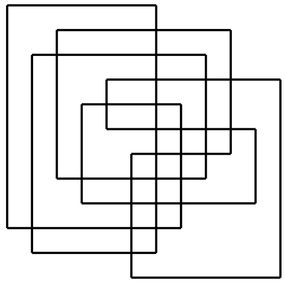

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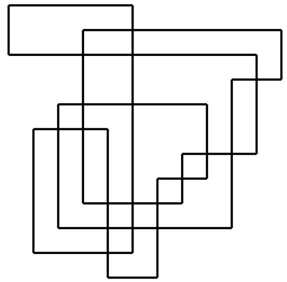

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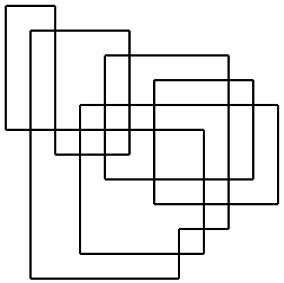

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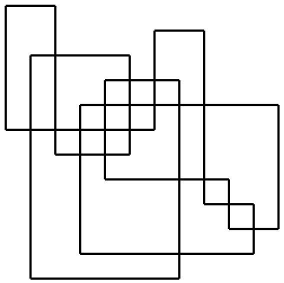

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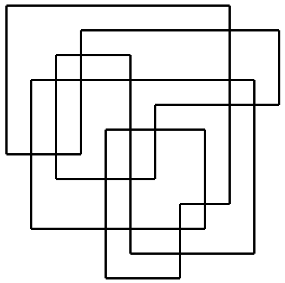

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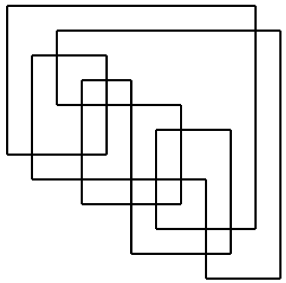

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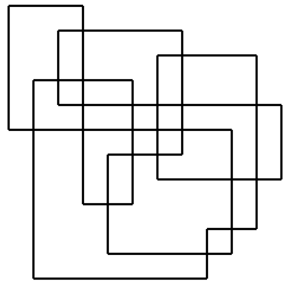

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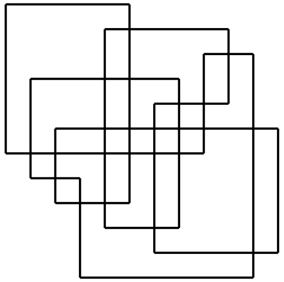

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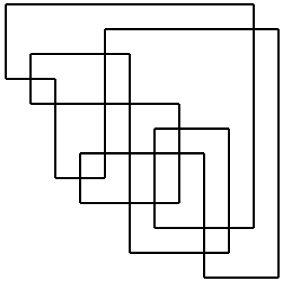

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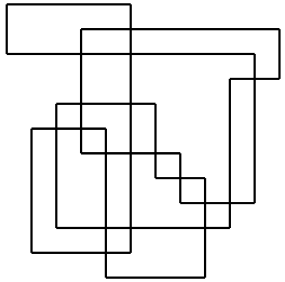

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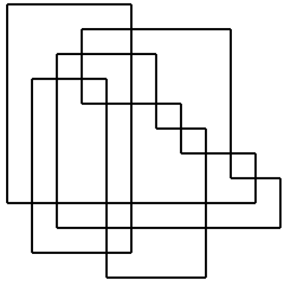

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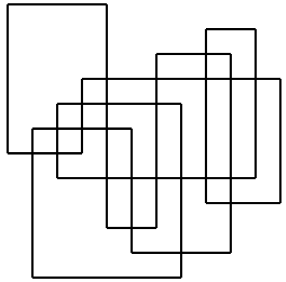

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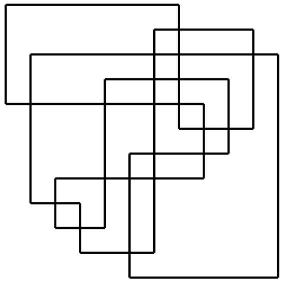

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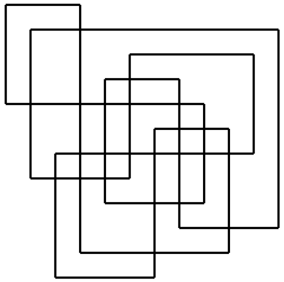

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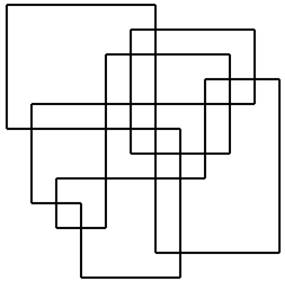

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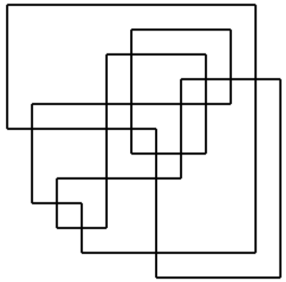

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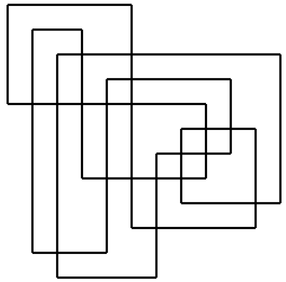

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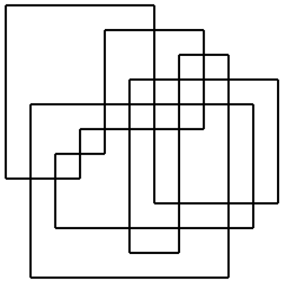

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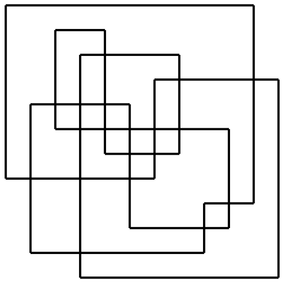

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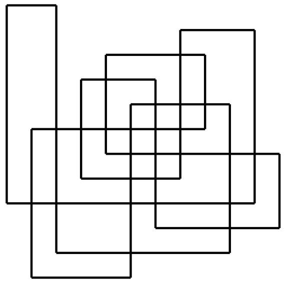

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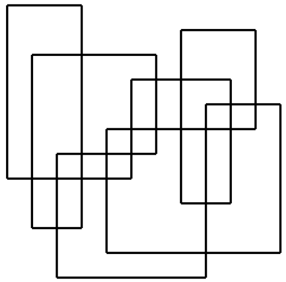

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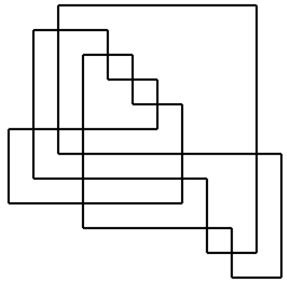

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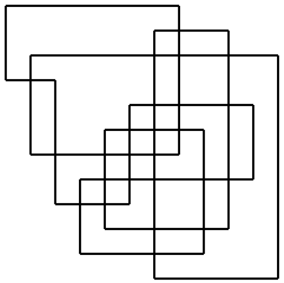

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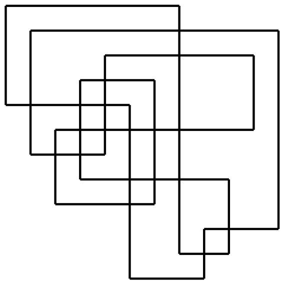

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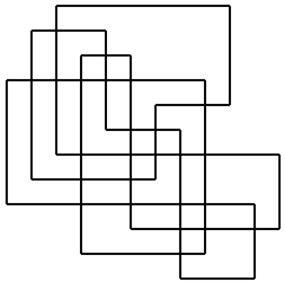

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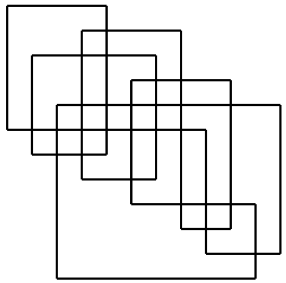

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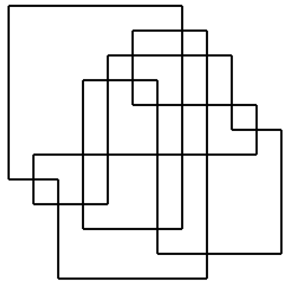

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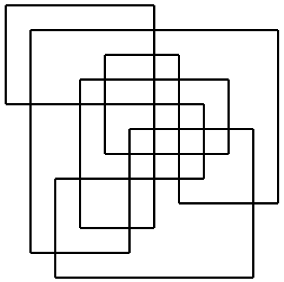

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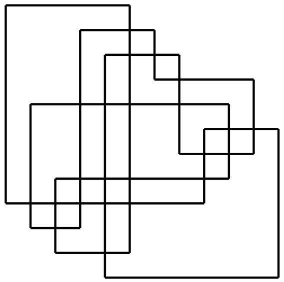

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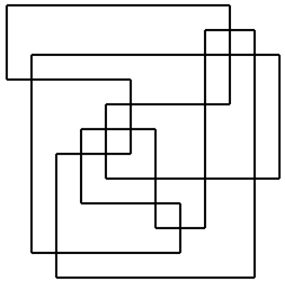

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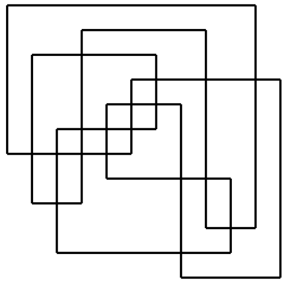

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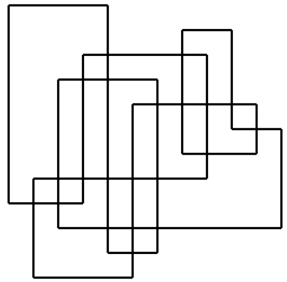

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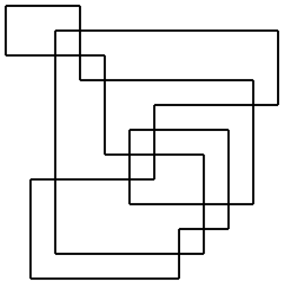

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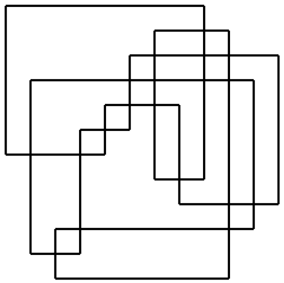

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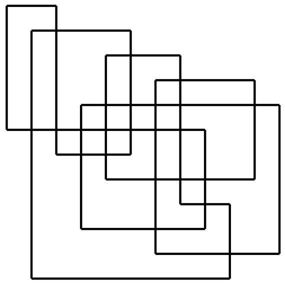

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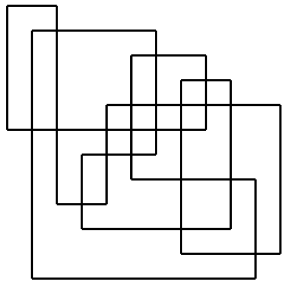

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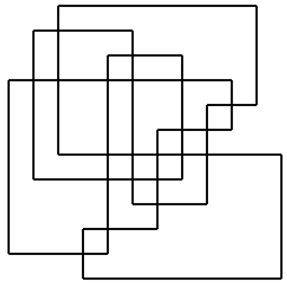

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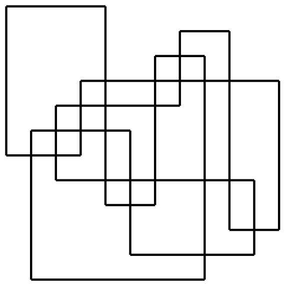
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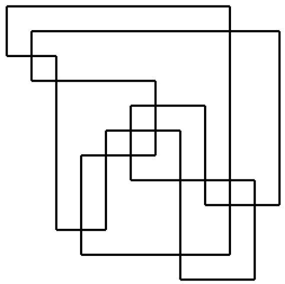
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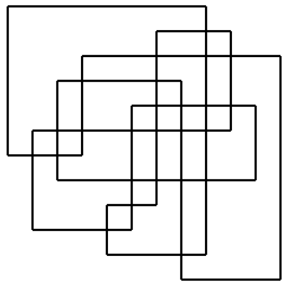
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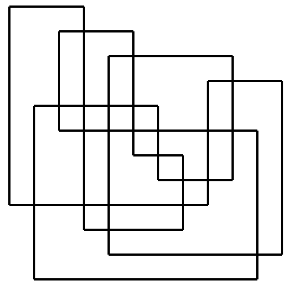
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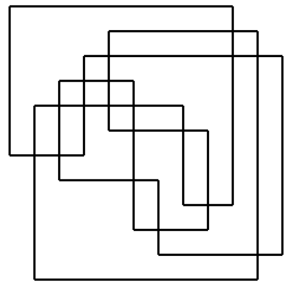
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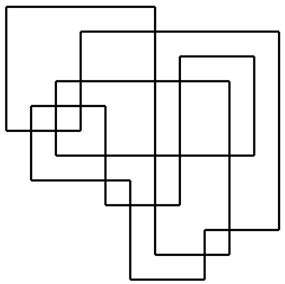
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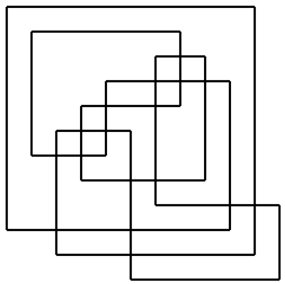
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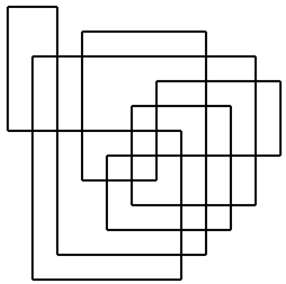
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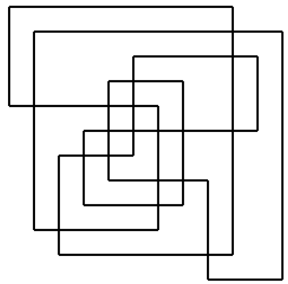
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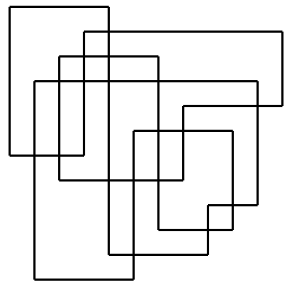
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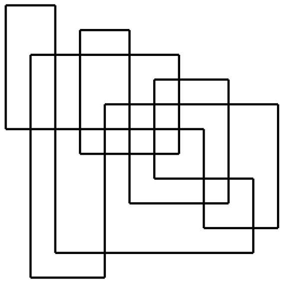

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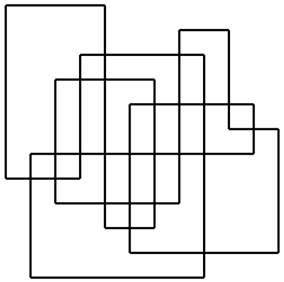

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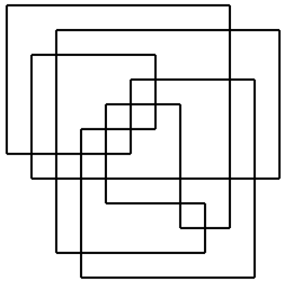

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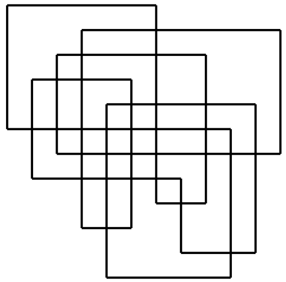

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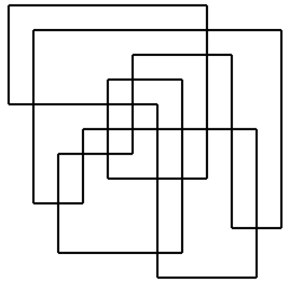

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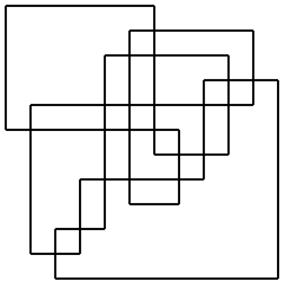

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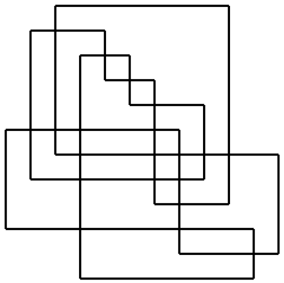

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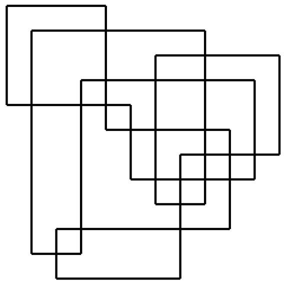

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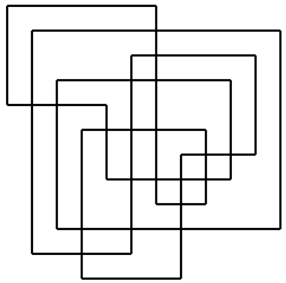

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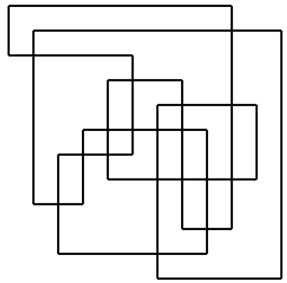

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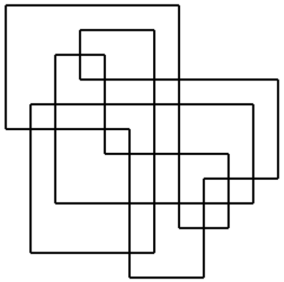

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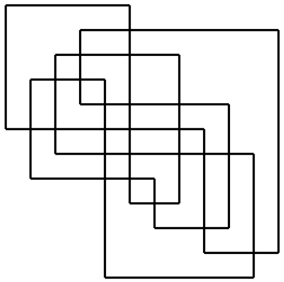

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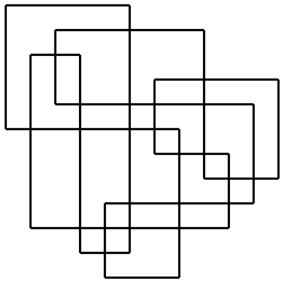

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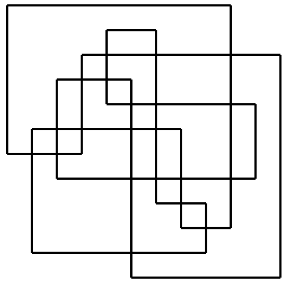

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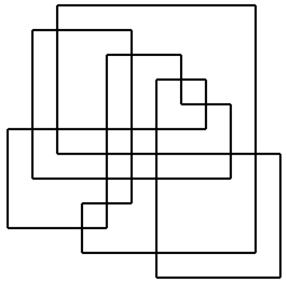

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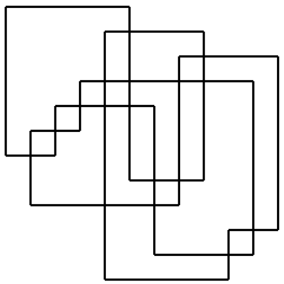

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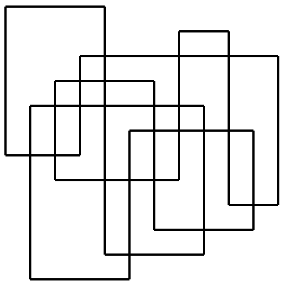

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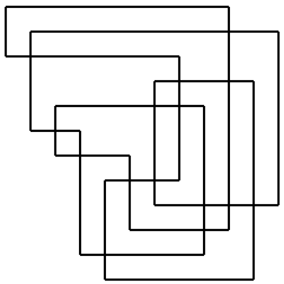

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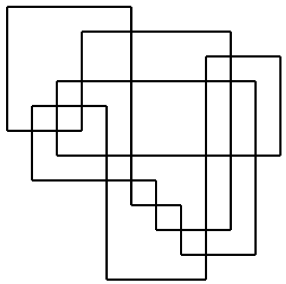

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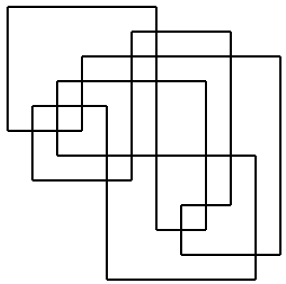

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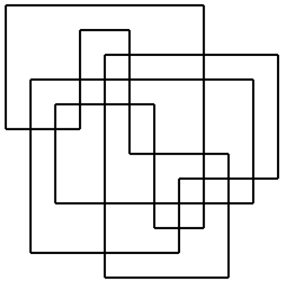

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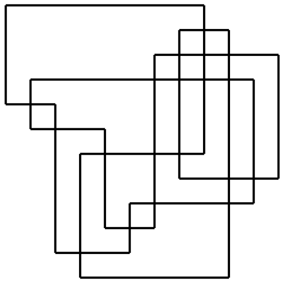

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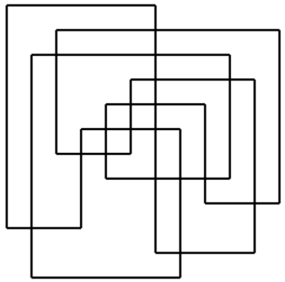

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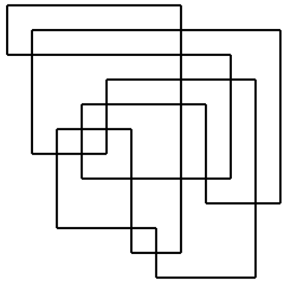

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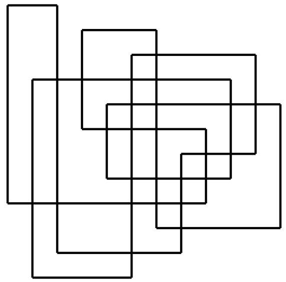

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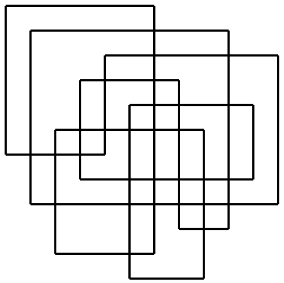

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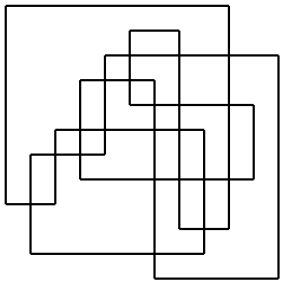

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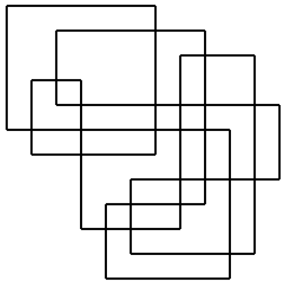

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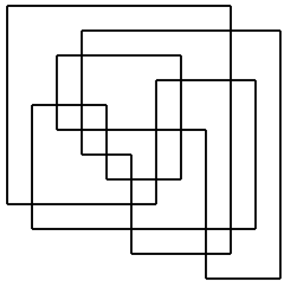

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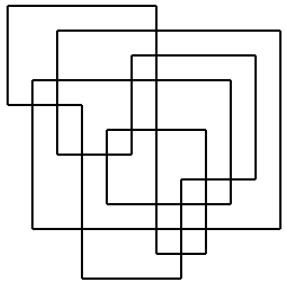

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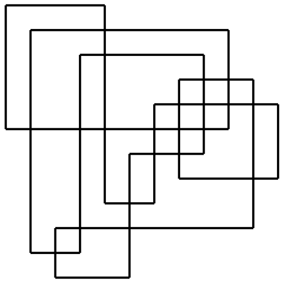

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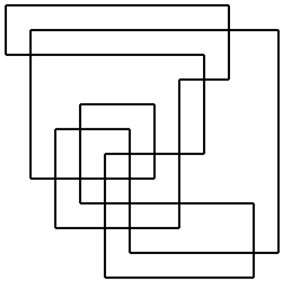

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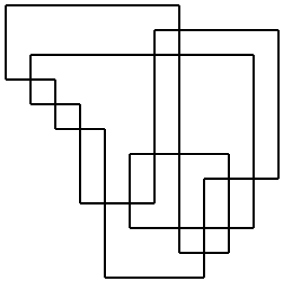

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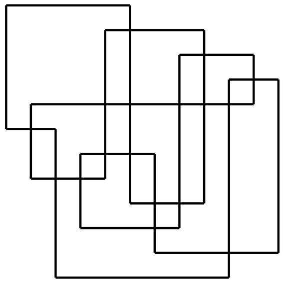

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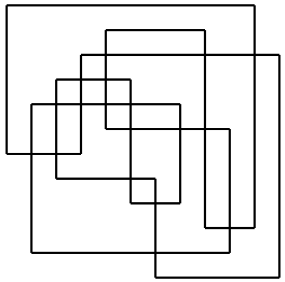

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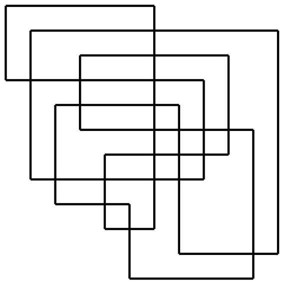

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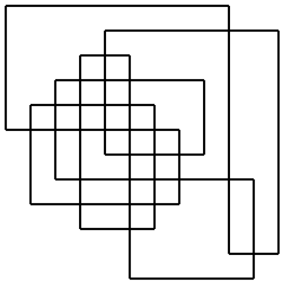

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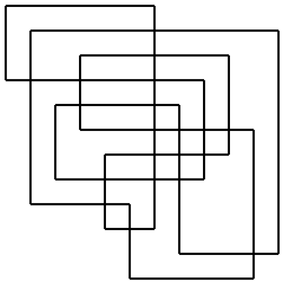

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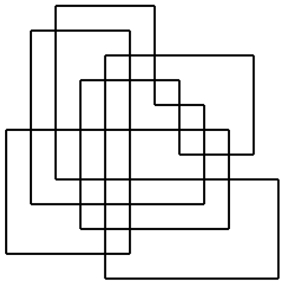

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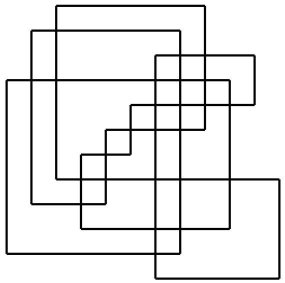

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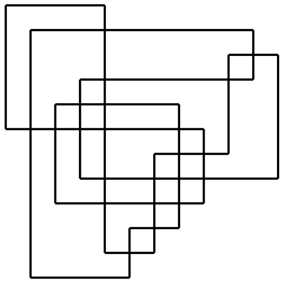

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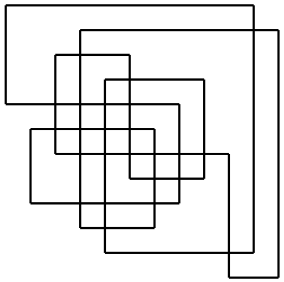

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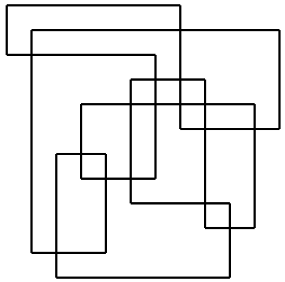

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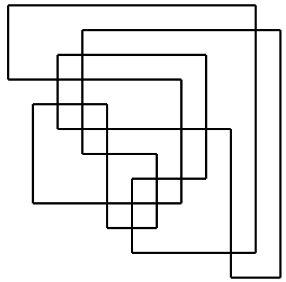

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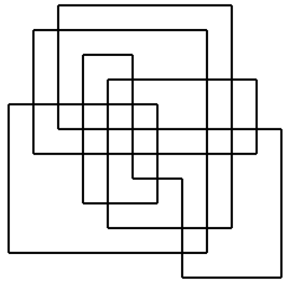

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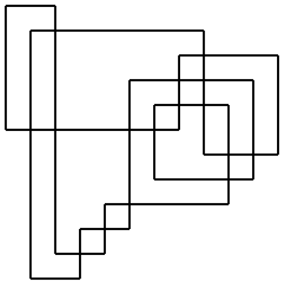

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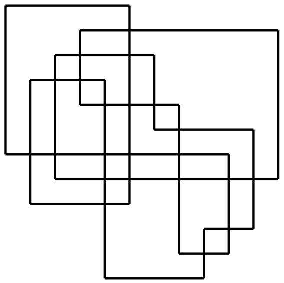

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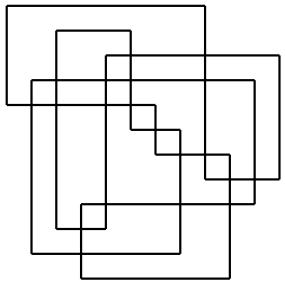

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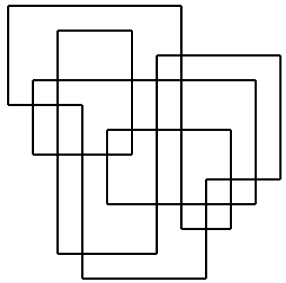

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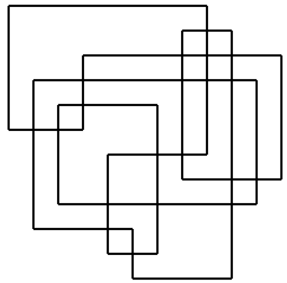

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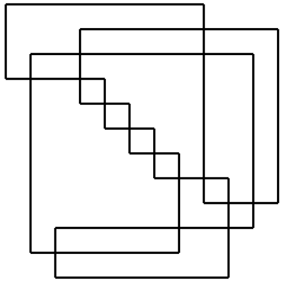

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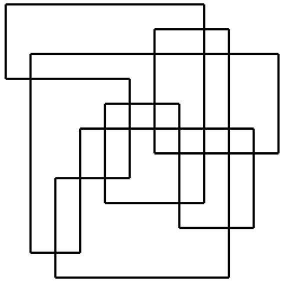

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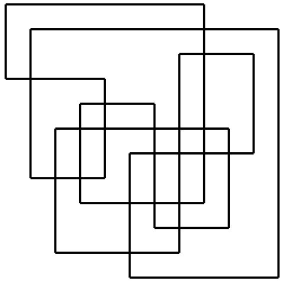

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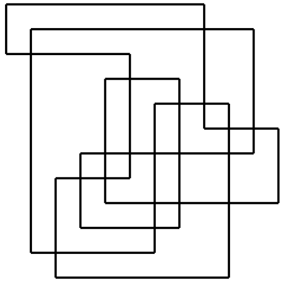

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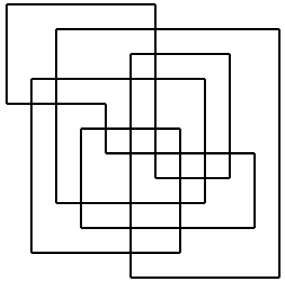

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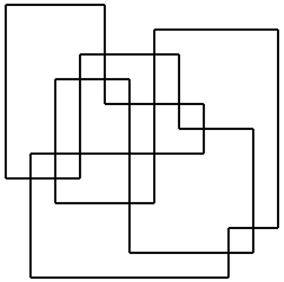

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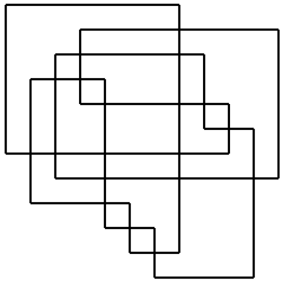

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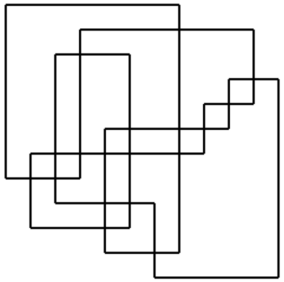

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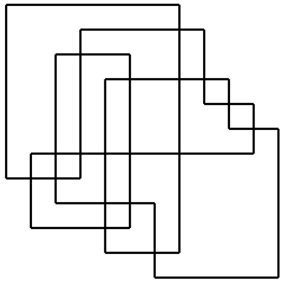

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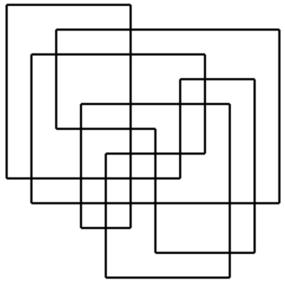

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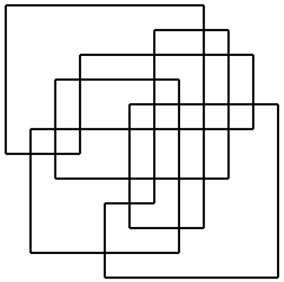
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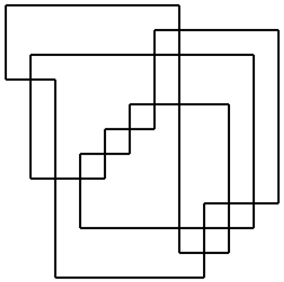
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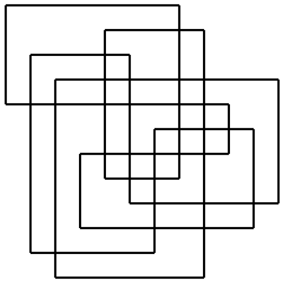
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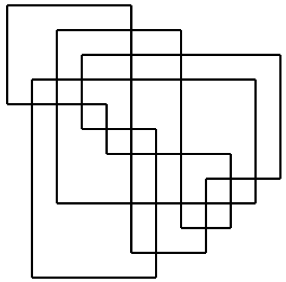
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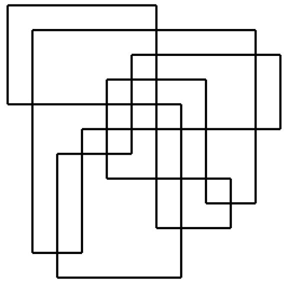
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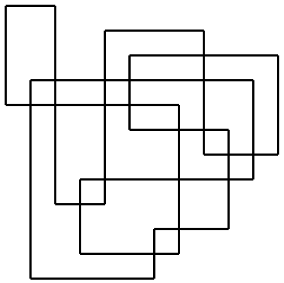
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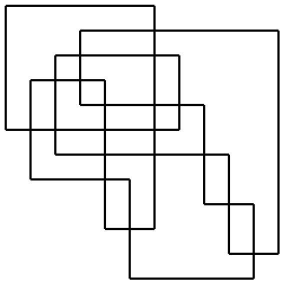
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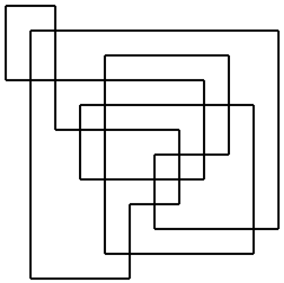
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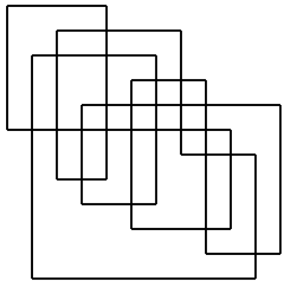
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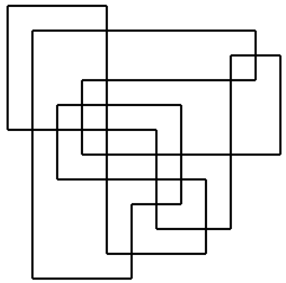
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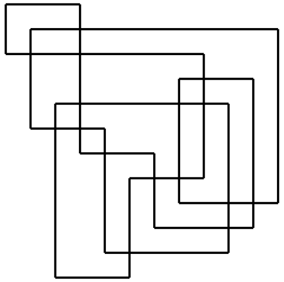

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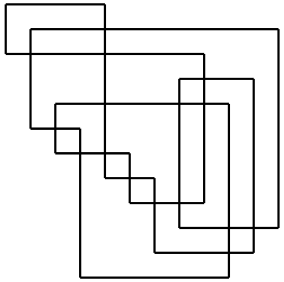

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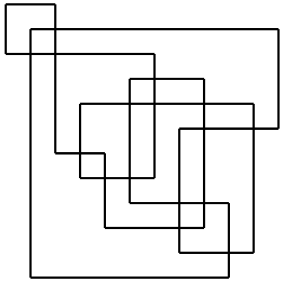

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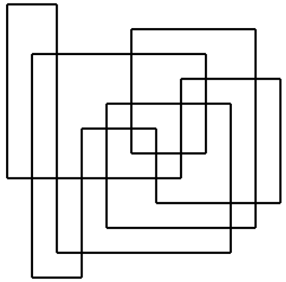

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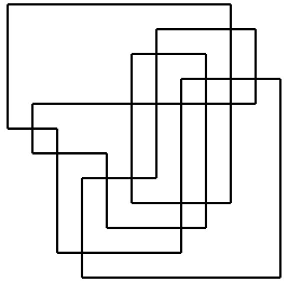

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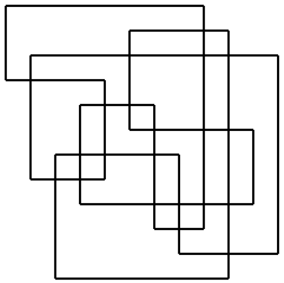

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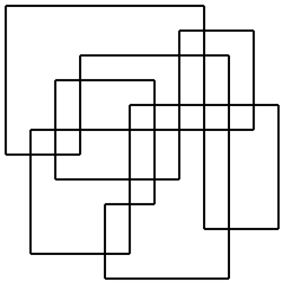

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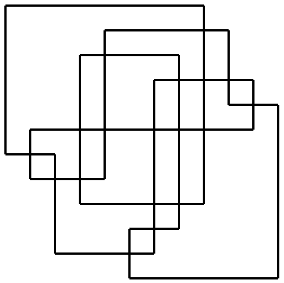

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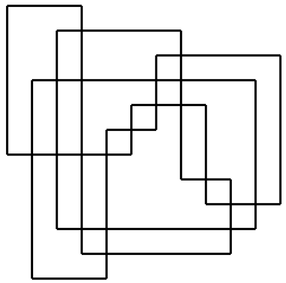

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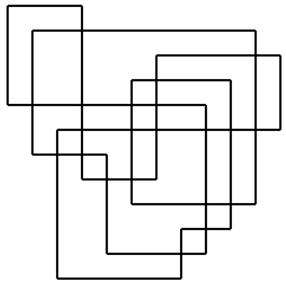

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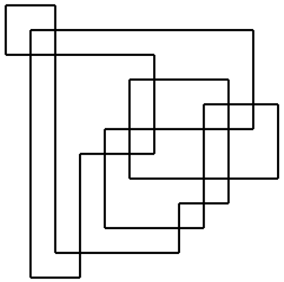

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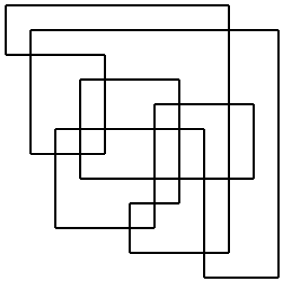

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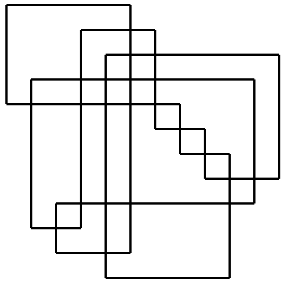

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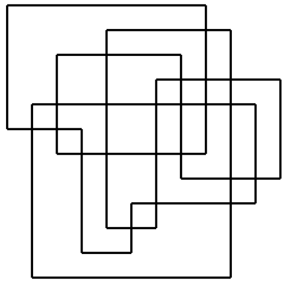

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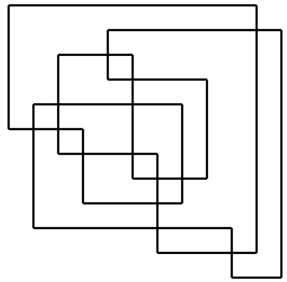

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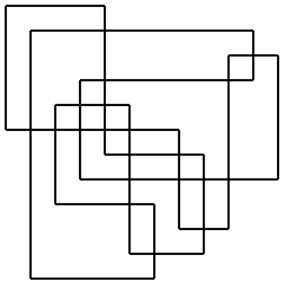

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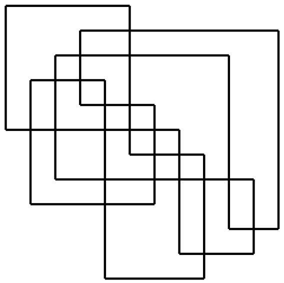

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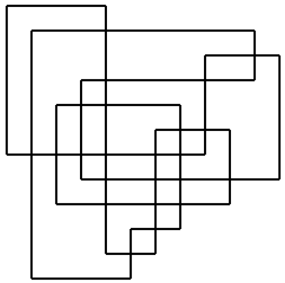

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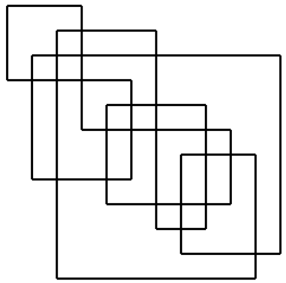

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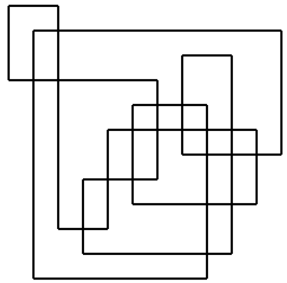

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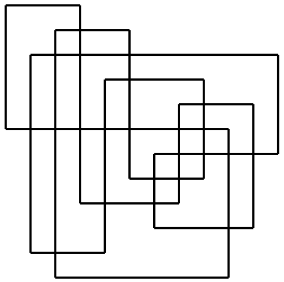

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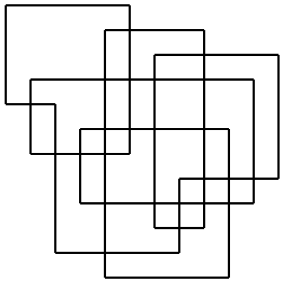

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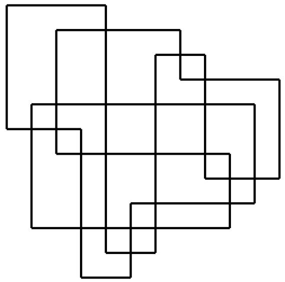

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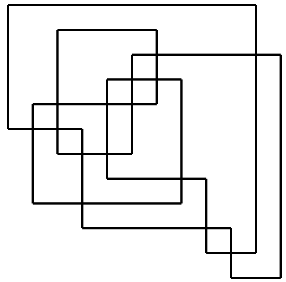

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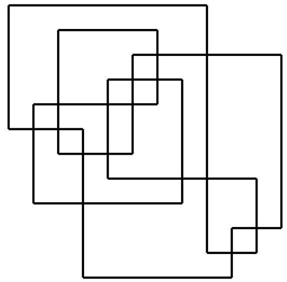

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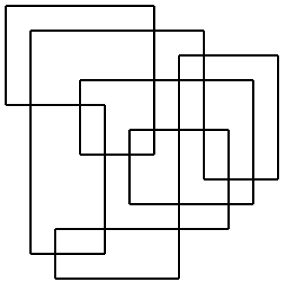

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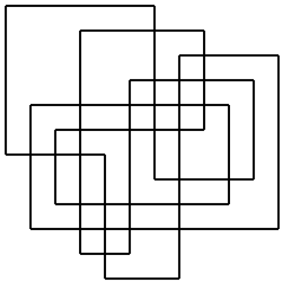

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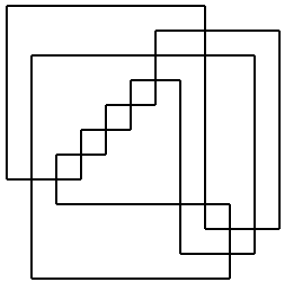

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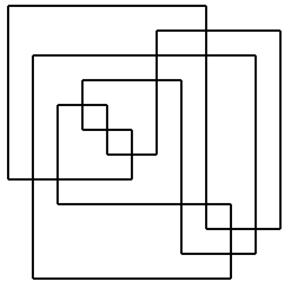

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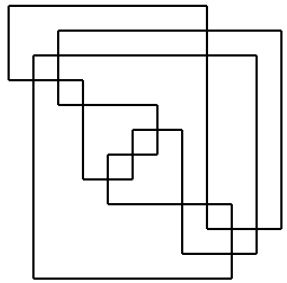

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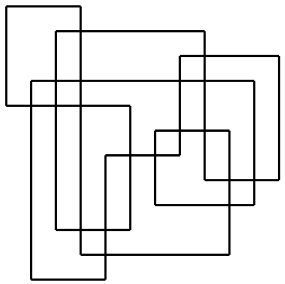

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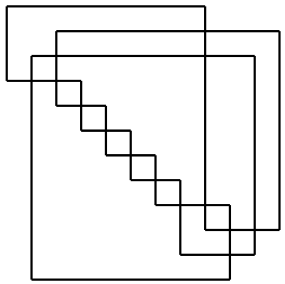

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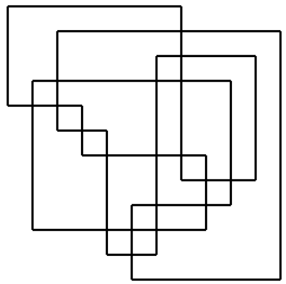

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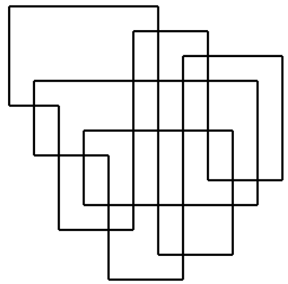

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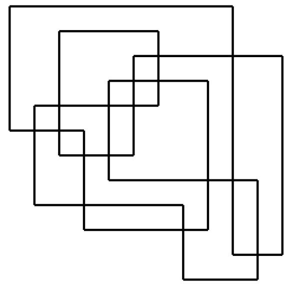

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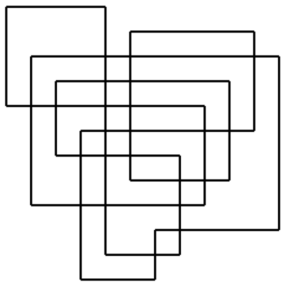

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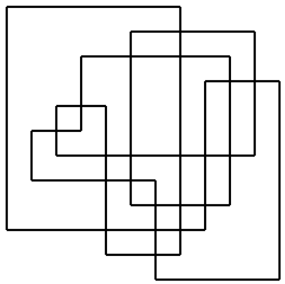

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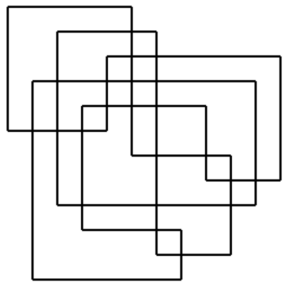

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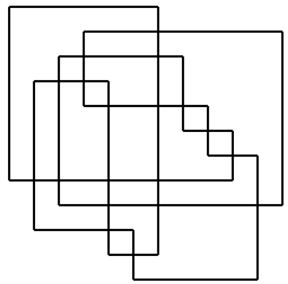

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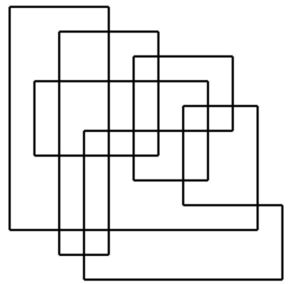

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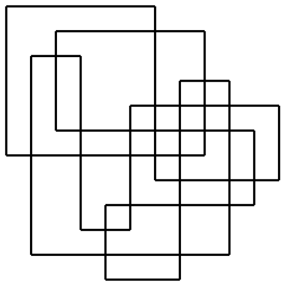

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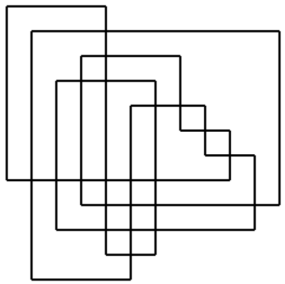

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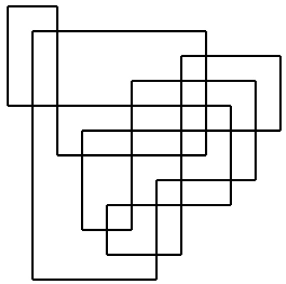

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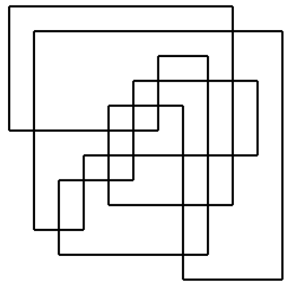
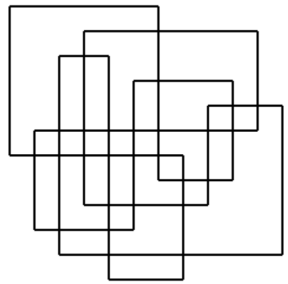


$13n3600$



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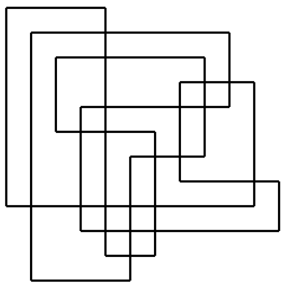

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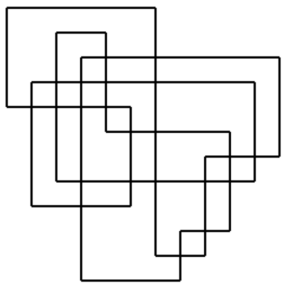

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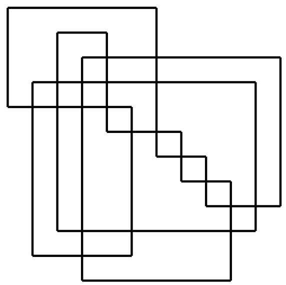

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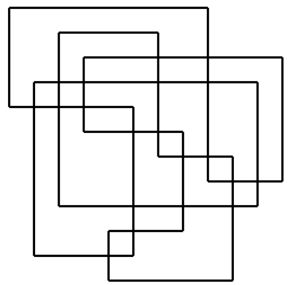

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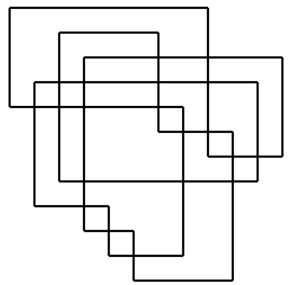

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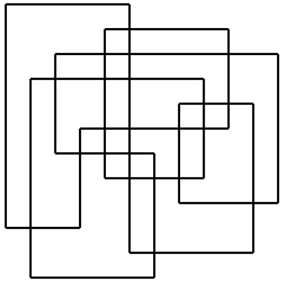

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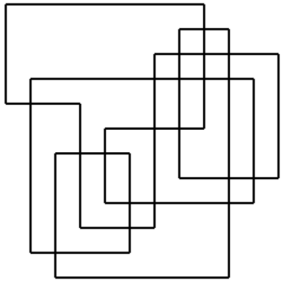

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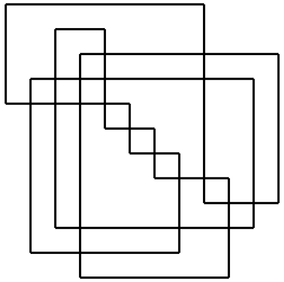

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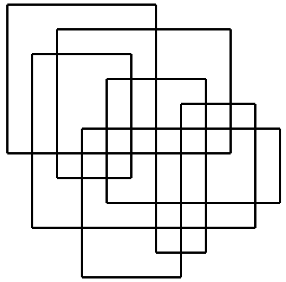

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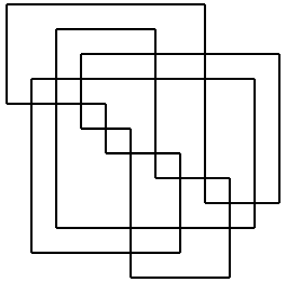

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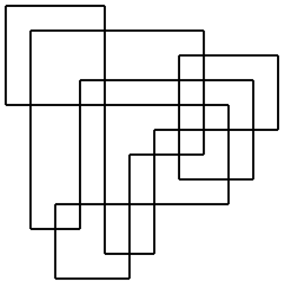
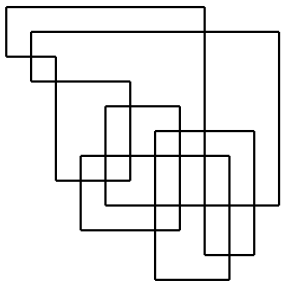
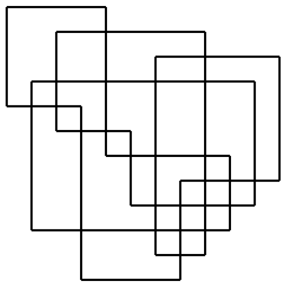


13n3698

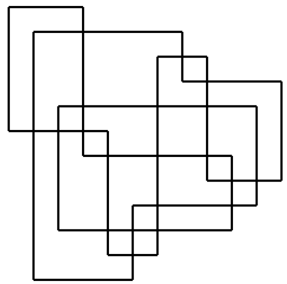
Examples related to Theorem A (knots with arc index 12)



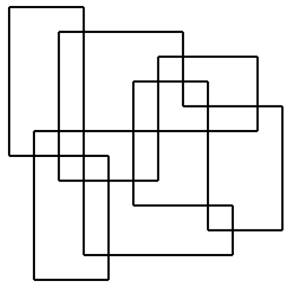
$13n2204$



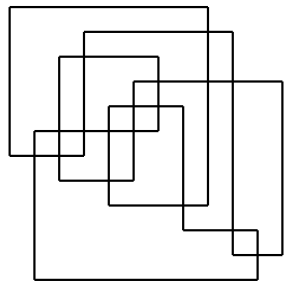
$13n3051$



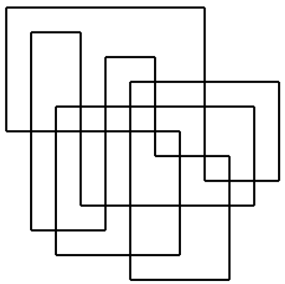
$13n3070$



$13n3401$



$13n3680$



$13n3701$

An Example Theorem B

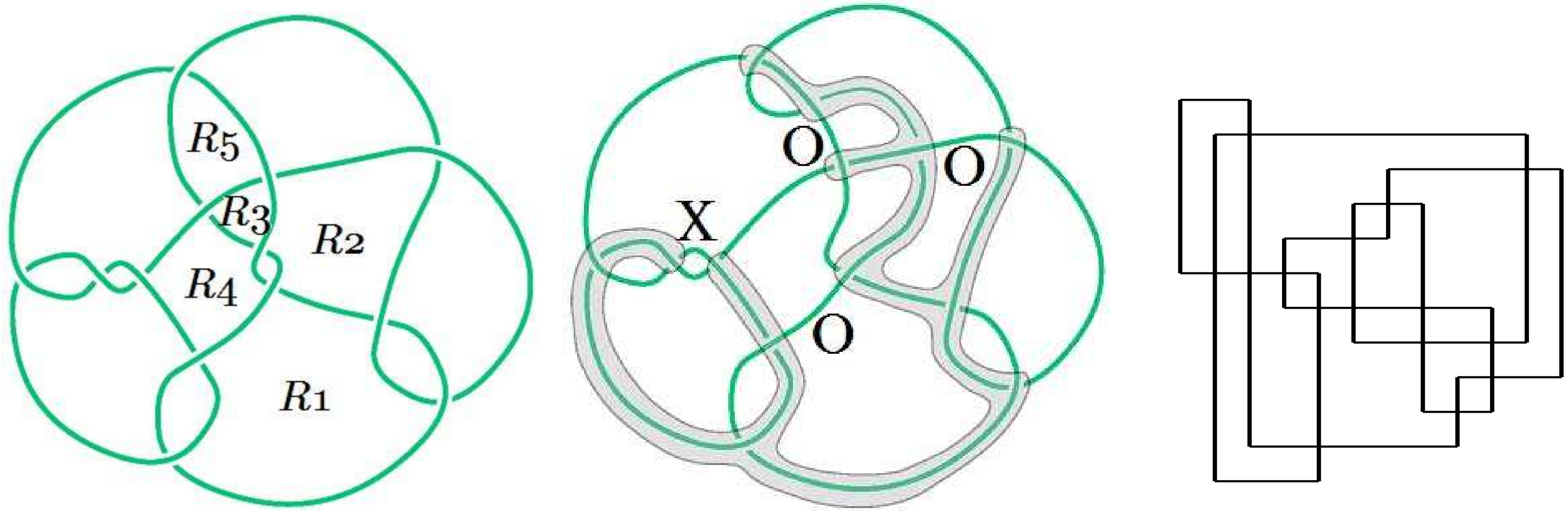
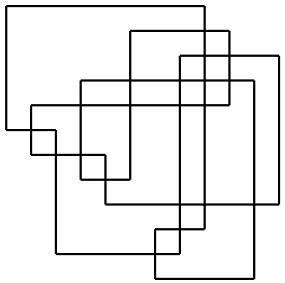
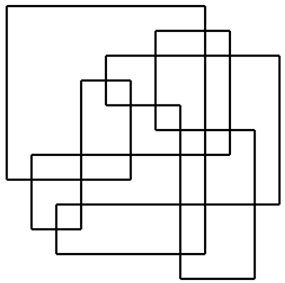


Figure 21: 2-nonalternating diagram : $13n2942$

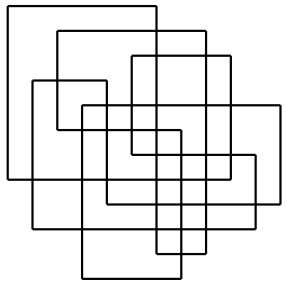
Examples of Theorem B (knots with arc index 12)



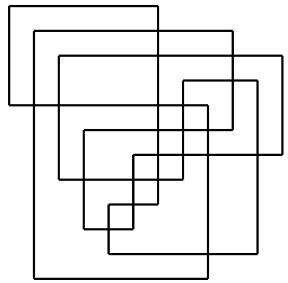
$13n1221$



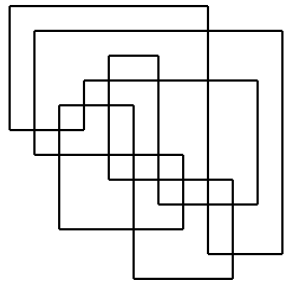
$13n1252$



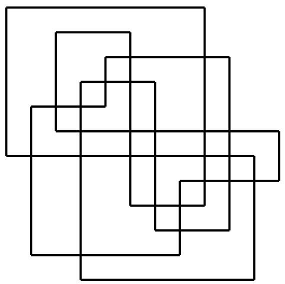
$13n1558$



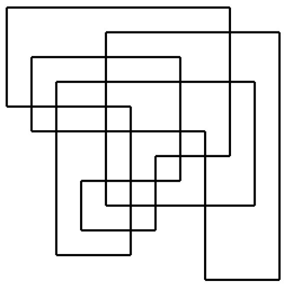
$13n1943$



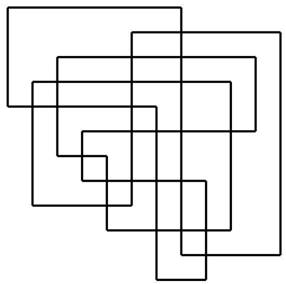
$13n2053$



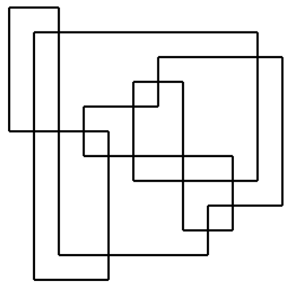
$13n2174$



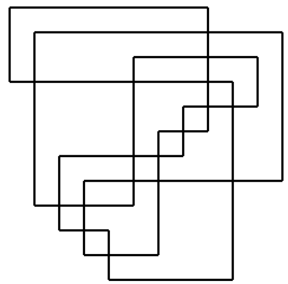
$13n2433$



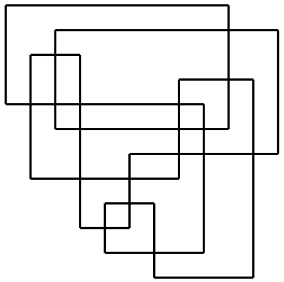
$13n2473$



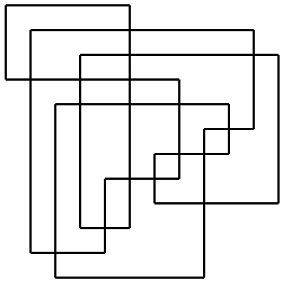
$13n2942$



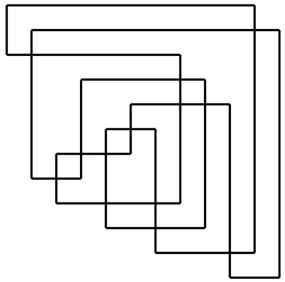
$13n3180$



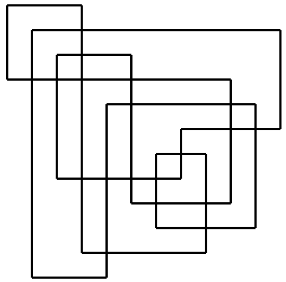
$13n3359$



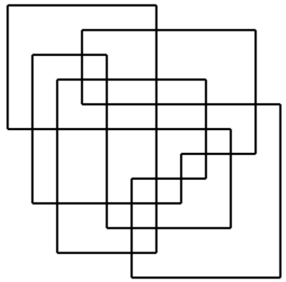
$13n3482$



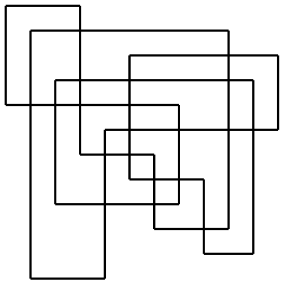
$13n3529$



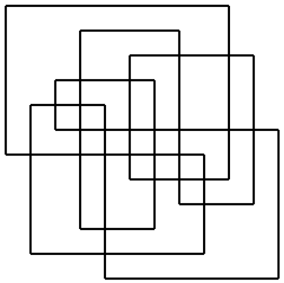
$13n3568$



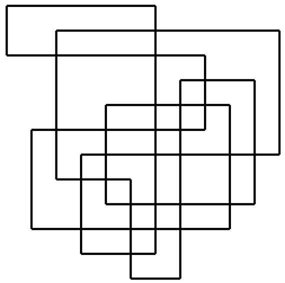
$13n3700$



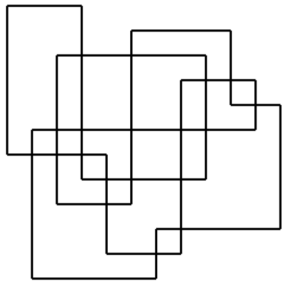
$13n4159$



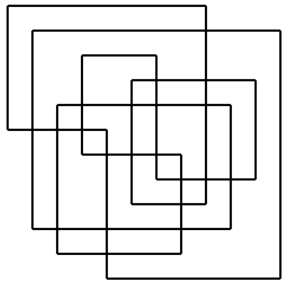
$13n4177$



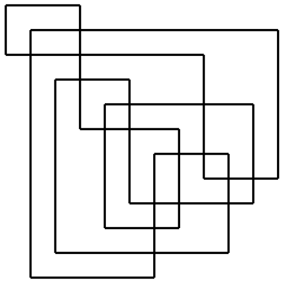
$13n4202$



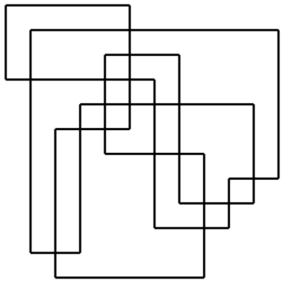
$13n4229$



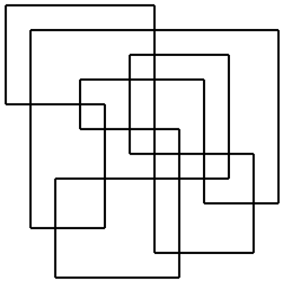
$13n4231$



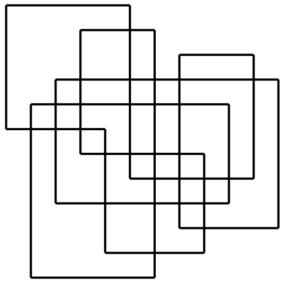
13n4235



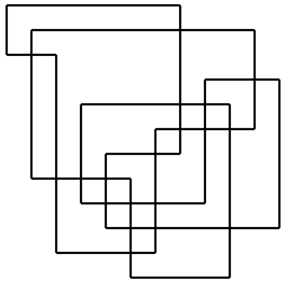
13n4236



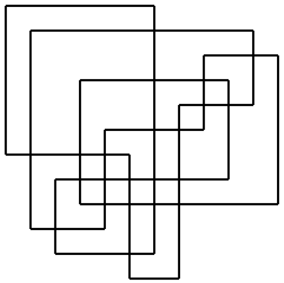
13n4258



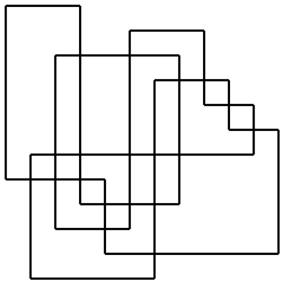
13n4276



13n4333



13n4336



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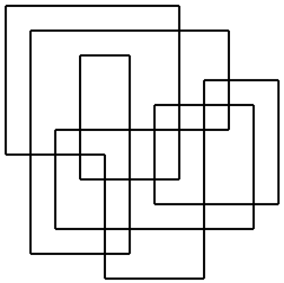
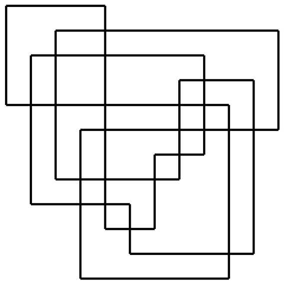
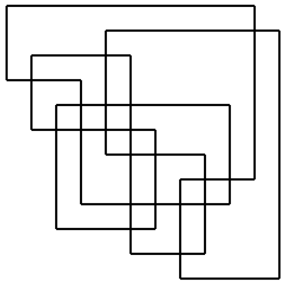


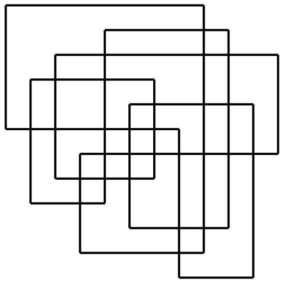
13n4375



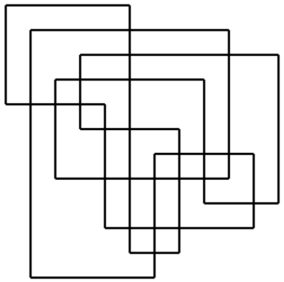
13n4379



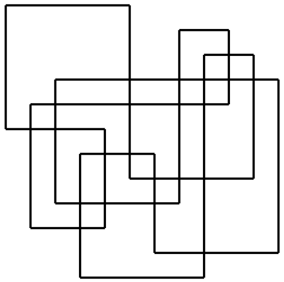
13n4573



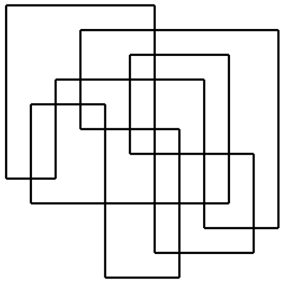
13n4588



13n4797



13n4800



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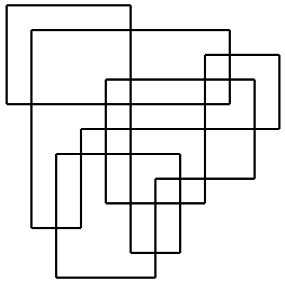
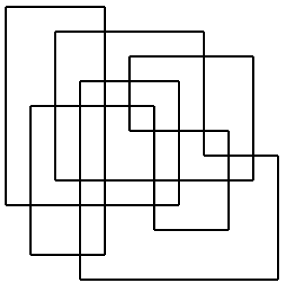
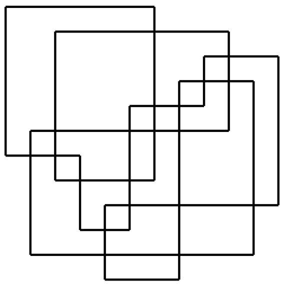


13n4894



13n4895



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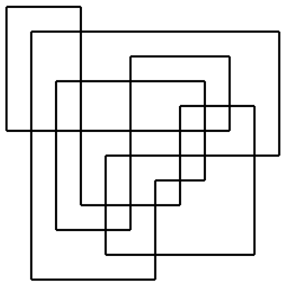
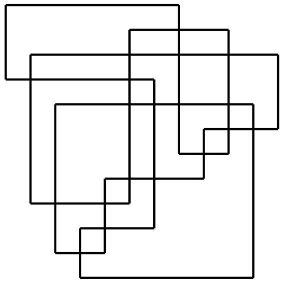
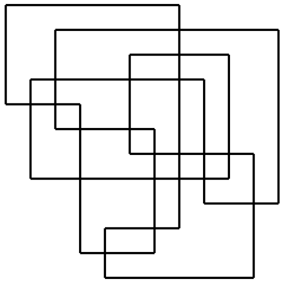


13n5095

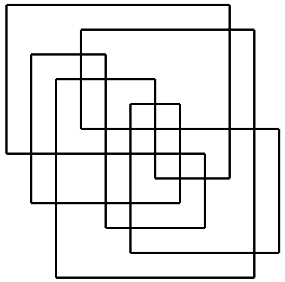
Examples related to Theorem B (knots with arc index 12)



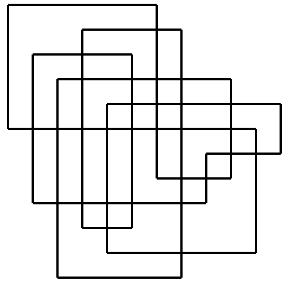
$13n1824$



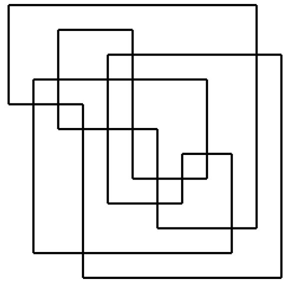
$13n1933$



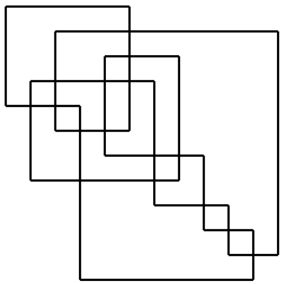
$13n3047$



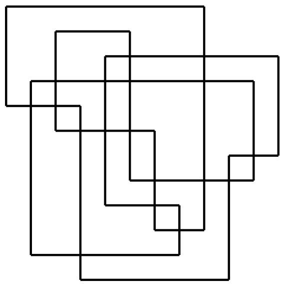
$13n3475$



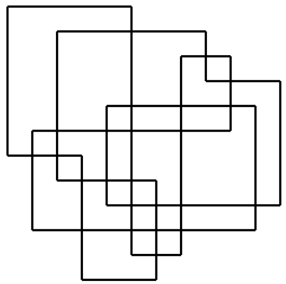
$13n4308$



$13n4386$



$13n4467$



$13n4580$

An Example of Theorem C

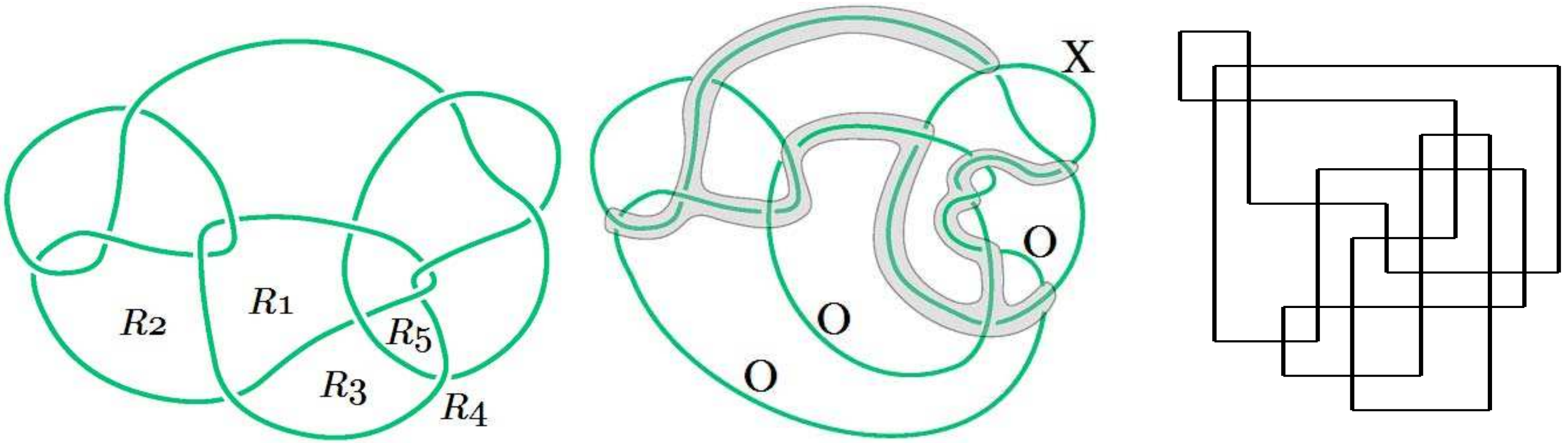
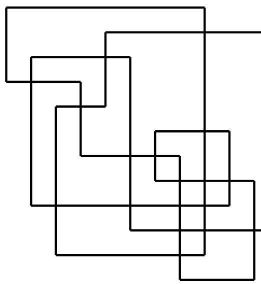
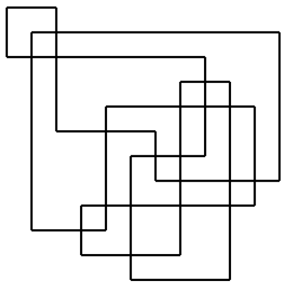


Figure 22: Almost alternating diagram : $13n0635$

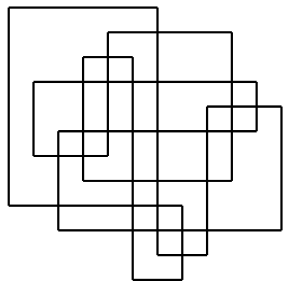
Examples of Theorem C (knots with arc index 12)



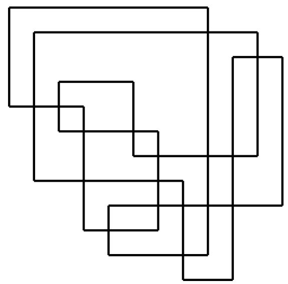
13n613



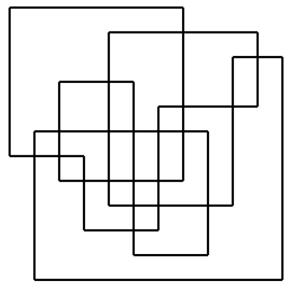
13n635



13n649



13n714



13n4031

Knots whose arc index equals crossing number (1)

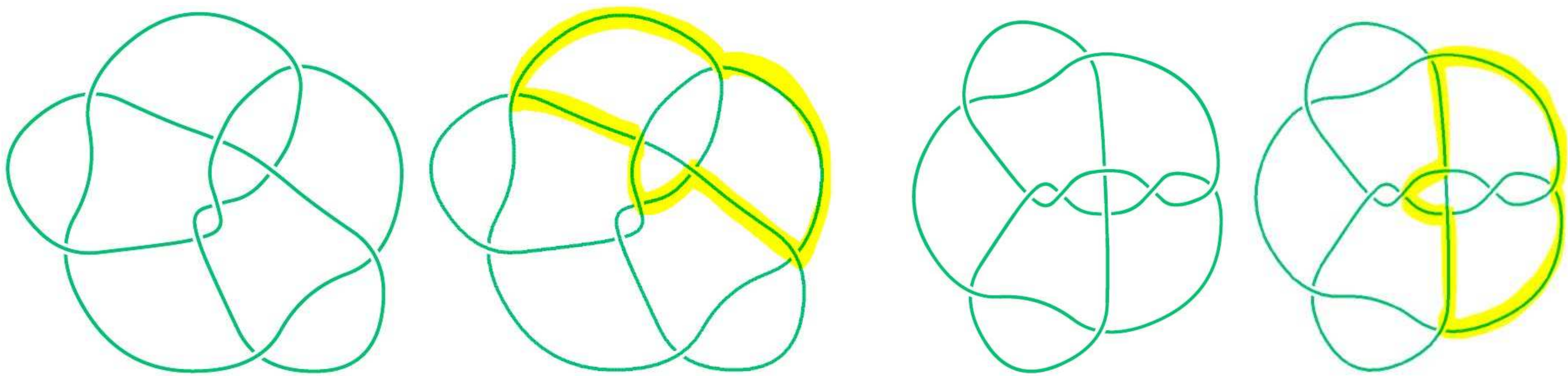


Figure 23: $\alpha(9n8) = 9$, $\alpha(10n41) = 10$

Knots whose arc index equals crossing number (2)

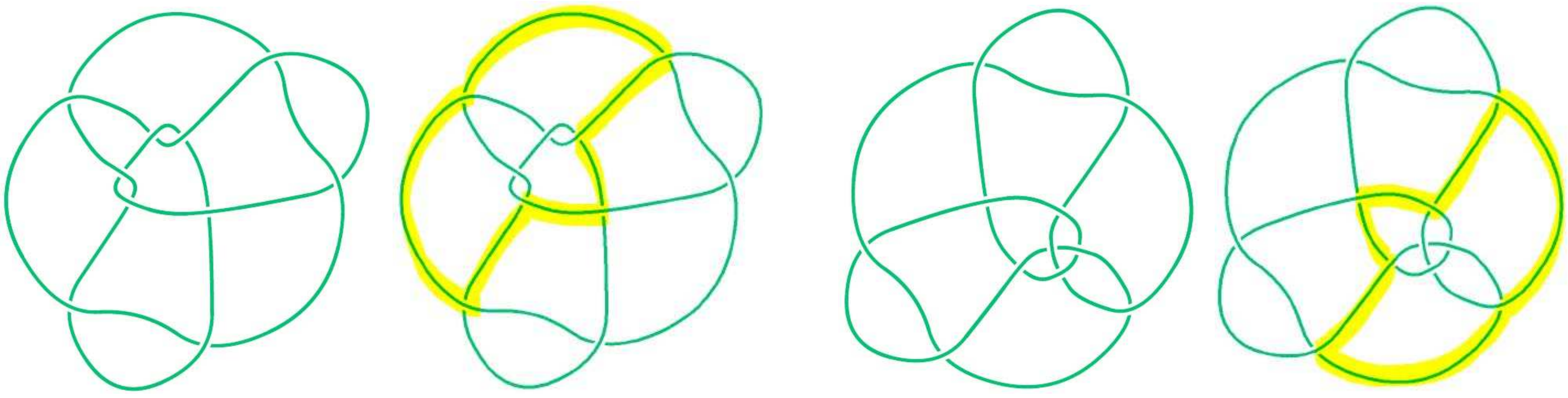


Figure 24: $\alpha(10n42) = 10$, $\alpha(11n163) = 11$

Knots whose arc index equals crossing number (3)

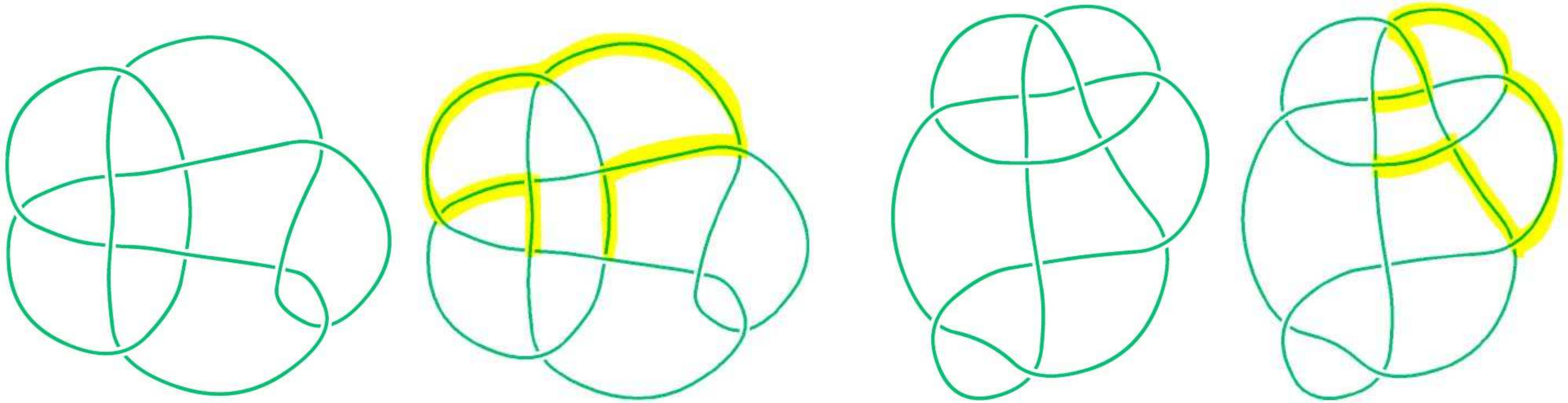


Figure 25: $\alpha(10n24) = 10$, $\alpha(11n85) = 11$

Knots whose arc index equals crossing number (4)

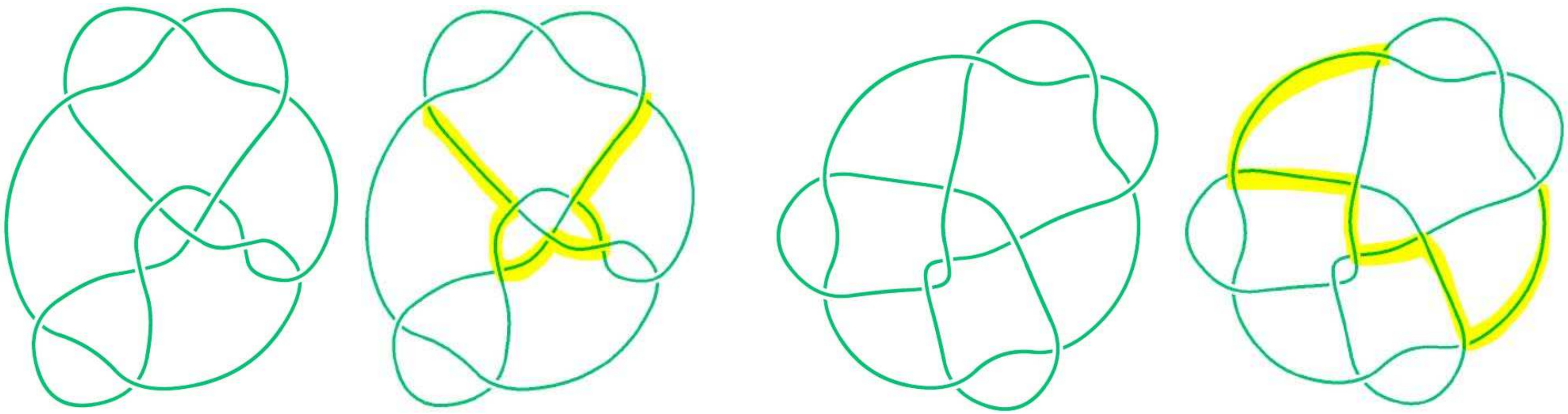


Figure 26: $\alpha(11n113) = 11$, $\alpha(11n169) = 11$

Knots whose arc index equals crossing number (5)

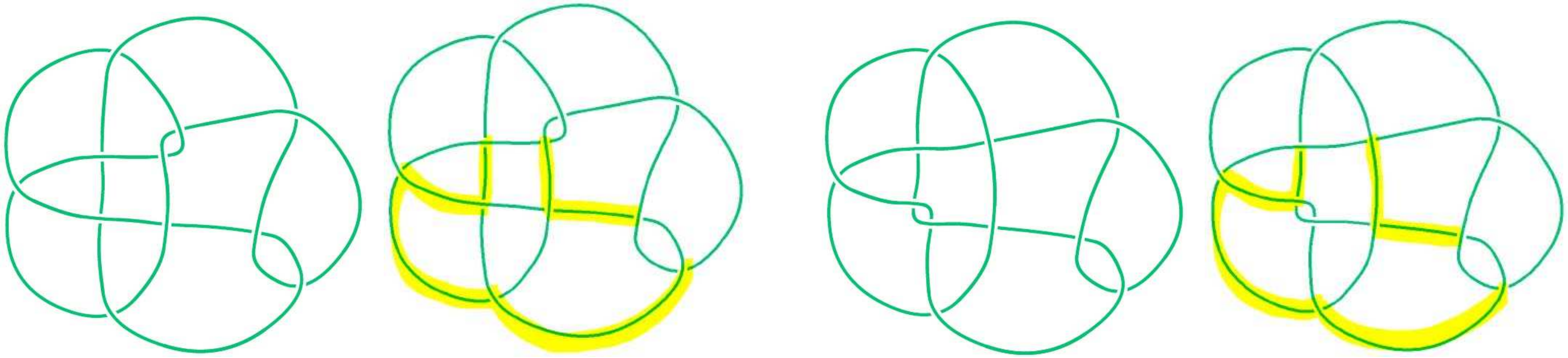


Figure 27: $\alpha(11n93) = 11$, $\alpha(11n124) = 11$

Knots whose arc index equals crossing number (6)

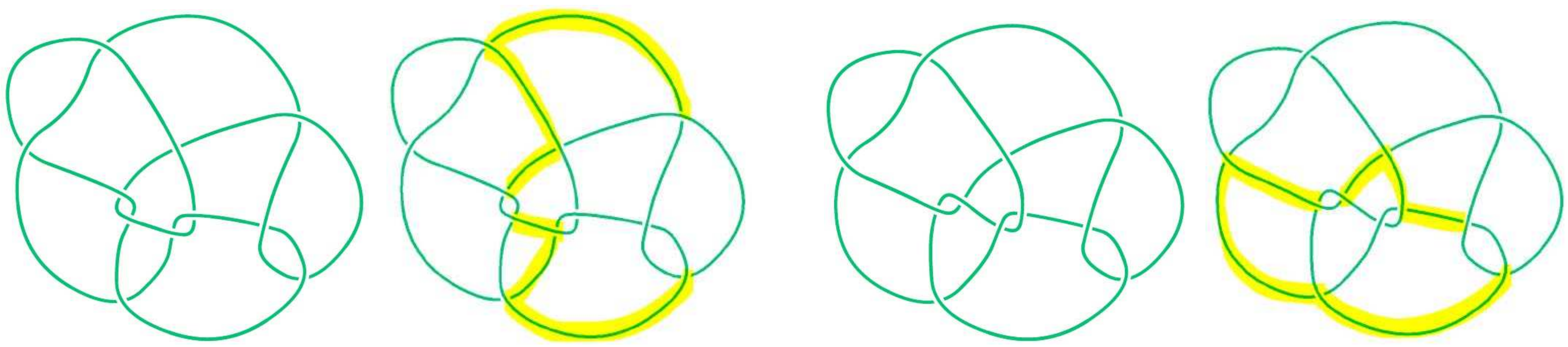


Figure 28: $\alpha(11n121) = 11$, $\alpha(11n127) = 11$

Thank you very much.