A Sufficient Condition for Intrinsic Linking [1]

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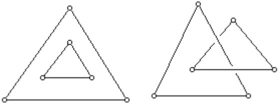
August 19, 2010

[1] Joint work with Jesse Campbell, Thomas Mattman, Joel Pyzer, Matt Rodrigues, and Sam Williams

An embedding of a graph is **linked** if it contains a non-trivial link



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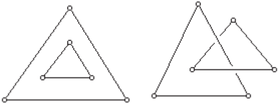

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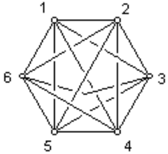

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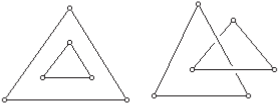

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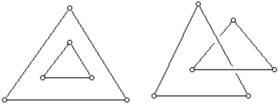

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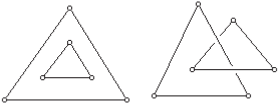

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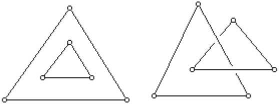

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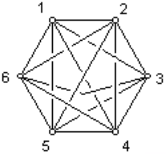

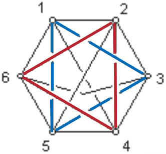

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Intrinsic linking and knotting are measures of complexity

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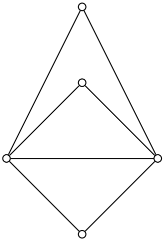

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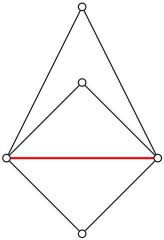

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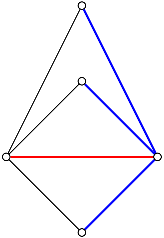

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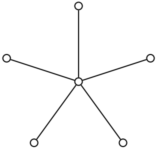

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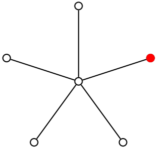

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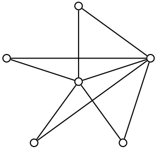

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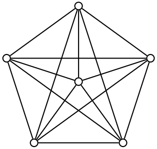

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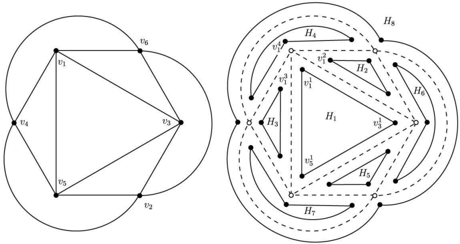

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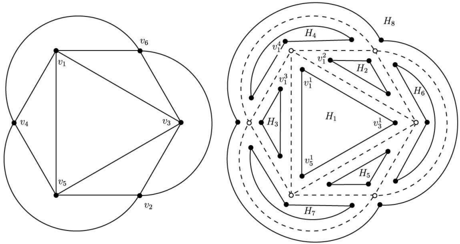

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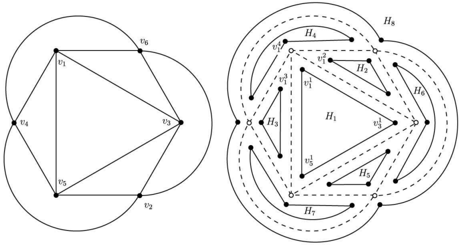

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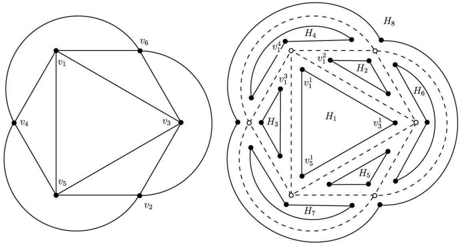

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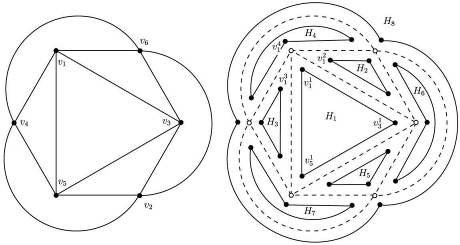

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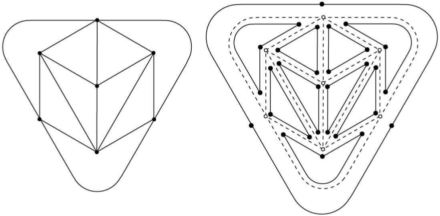

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Lemma (Blain et. al. and Ozawa et. al.)

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knotted	7	5,5	3,3,3 4,3,2 4,4,1	3,2,2,2 4,2,2,1 3,3,2,1 3,3,1,1	2,2,2,2,1 3,2,2,1,1 3,2,1,1,1	2,2,1,1,1,1 3,1,1,1,1,1	All
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Table: Intrinsic knotting of k-partite graphs. (Blain et. al.)

k	1	2	3	4	5	6	≥ 7
linked	7-e	4,4-e	4,3,1-e 3,3,2-e 4,2,2-e	2,2,2,2-e 3,2,1,1-(b,c) 4,2,1,1-e 3,3,1,1-e 3,2,2,1-e	2,2,1,1,1-(b,c) 3,1,1,1,1-(b,c) 4,1,1,1,1-e 3,2,1,1,1-e 2,2,2,1,1-e	2,1,1,1,1,1-e	All
not linked	6-e	n,3-e	3,2,2-e n,2,1-e 3,3,1-e	2,2,2,1-e n,1,1,1-e 3,2,1,1-(a,b) 3,2,1,1-(a,c) 3,2,1,1-(c,d)	2,2,1,1,1-(a,b) 2,2,1,1,1-(c,d) 3,1,1,1,1-(a,b) 2,1,1,1,1-e	1,1,1,1,1,1-e	None

Table: Intrinsic Linking of 1 Deficient Graphs.

k	1	2	3	4	5	6	≥ 7
linked	7-e	4,4-e	4,3,1-e 3,3,2-e 4,2,2-e	2,2,2,2-e 3,2,1,1-(b,c) 4,2,1,1-e 3,3,1,1-e 3,2,2,1-e	2,2,1,1,1-(b,c) 3,1,1,1,1-(b,c) 4,1,1,1,1-e 3,2,1,1,1-e 2,2,2,1,1-e	2,1,1,1,1,1-e	All
not linked	6-e	n,3-e	3,2,2-e n,2,1-e 3,3,1-e	2,2,2,1-e n,1,1,1-e 3,2,1,1-(a,b) 3,2,1,1-(a,c) 3,2,1,1-(c,d)	2,2,1,1,1-(a,b) 2,2,1,1,1-(c,d) 3,1,1,1,1-(a,b) 2,1,1,1,1-e	1,1,1,1,1,1-e	None

Table: Intrinsic Linking of 1 Deficient Graphs.

k	1	2	3	4	5	6	≥ 7
linked	7-e	4,4-e	4,3,1-e 3,3,2-e 4,2,2-e	2,2,2,2-e 3,2,1,1-(b,c) 4,2,1,1-e 3,3,1,1-e 3,2,2,1-e	2,2,1,1,1-(b,c) 3,1,1,1,1-(b,c) 4,1,1,1,1-e 3,2,1,1,1-e 2,2,2,1,1-e	2,1,1,1,1,1-e	All
not linked	6-e	*n*,3-e	3,2,2-e *n*,2,1-e 3,3,1-e	2,2,2,1-e *n*,1,1,1-e 3,2,1,1-(a,b) 3,2,1,1-(a,c) 3,2,1,1-(c,d)	2,2,1,1,1-(a,b) 2,2,1,1,1-(c,d) 3,1,1,1,1-(a,b) 2,1,1,1,1-e	1,1,1,1,1,1-e	None

Table: Intrinsic Linking of 1 Deficient Graphs.

k	1	2	3	4	5	6	≥ 7
linked	7-e	4,4-e	4,3,1-e 3,3,2-e 4,2,2-e	2,2,2,2-e 3,2,1,1-(b,c) 4,2,1,1-e 3,3,1,1-e 3,2,2,1-e	2,2,1,1,1-(b,c) 3,1,1,1,1-(b,c) 4,1,1,1,1-e 3,2,1,1,1-e 2,2,2,1,1-e	2,1,1,1,1,1-e	All
not linked	6-e	*n*,3-e	3,2,2-e *n*,2,1-e 3,3,1-e	2,2,2,1-e *n*,1,1,1-e 3,2,1,1-(a,b) 3,2,1,1-(a,c) 3,2,1,1-(c,d)	2,2,1,1,1-(a,b) 2,2,1,1,1-(c,d) 3,1,1,1,1-(a,b) 2,1,1,1,1-e	1,1,1,1,1,1-e	None

Table: Intrinsic Linking of 1 Deficient Graphs.

k	1	2	3	4	5	6	≥ 7
linked	7-e	4,4-e	4,3,1-e 3,3,2-e 4,2,2-e	2,2,2,2-e 3,2,1,1-(b,c) 4,2,1,1-e 3,3,1,1-e 3,2,2,1-e	2,2,1,1,1-(b,c) 3,1,1,1,1-(b,c) 4,1,1,1,1-e 3,2,1,1,1-e 2,2,2,1,1-e	2,1,1,1,1,1-e	All
not linked	6-e	n,3-e	3,2,2-e n,2,1-e 3,3,1-e	2,2,2,1-e n,1,1,1-e 3,2,1,1-(a,b) 3,2,1,1-(a,c) 3,2,1,1-(c,d)	2,2,1,1,1-(a,b) 2,2,1,1,1-(c,d) 3,1,1,1,1-(a,b) 2,1,1,1,1-e	1,1,1,1,1,1-e	None

Table: Intrinsic Linking of 1 Deficient Graphs.

k	1	2	3	4	5	6	7	≥ 8
knotted	8-e	5,5-e	3,3,3-e 4,3,2-e 4,4,1-e	3,2,2,2-e 4,2,2,1-e 3,3,2,1-e 4,3,1,1-e	2,2,2,2,1-e 3,2,1,1,1-(b,c) 4,2,1,1,1-e 3,3,1,1,1-e 3,2,2,1,1-e	2,2,1,1,1,1-(b,c) 3,1,1,1,1,1-(b,c) 3,2,1,1,1,1-e 2,2,2,1,1,1-e 4,1,1,1,1,1-e	2,1,1,1,1,1,1-e	All
not knotted	7-e	n,4-e	3,3,2-e n,2,2-e n,3,1-e	3,3,1,1-e 2,2,2,2-e 3,2,2,1-e n,2,1,1-e	3,2,1,1,1-(c,d) 3,2,1,1,1-(a,b) 3,2,1,1,1-(a,c) 2,2,2,1,1-e n,1,1,1,1-e	2,2,1,1,1,1-(a,b) 2,2,1,1,1,1-(c,d) 3,1,1,1,1,1-(a,b) 2,1,1,1,1,1-e	1,1,1,1,1,1,1-e	None

Table: Intrinsic knotting of 1 deficient graphs.

Thank You