142857の秘密

――数楽から数学へ――

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142857 の秘密

$142857 \times 1 = 142857$

$142857 \times 2 = 285714$

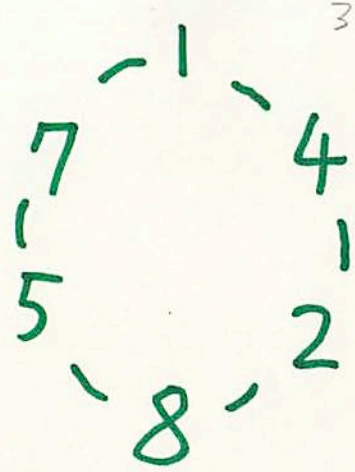
$142857 \times 3 = 428571$

$142857 \times 4 = 571428$

$142857 \times 5 = 714285$

$142857 \times 6 = 857142$

$142857 \times 7 = 999999$



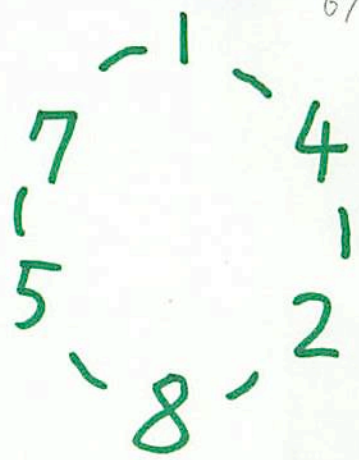
$\frac{1}{7} = 0.\dot{1}4285\dot{7}$

$= 0.142857142857142857\ldots\ldots$

$$
\begin{array}{r}
0.142857 \\
7\overline{)1} \\
\underline{1} \\
10 \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
1
\end{array}
$$

$$
\begin{array}{r}
0.142857 \\
7\overline{)\,1} \\
\hline
10 \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
1
\end{array}
$$

$$
\begin{array}{r}
0.285714 \\
7\overline{)\,2} \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
10 \\
7 \\
\hline
30 \\
28 \\
\hline
2
\end{array}
$$



$$\frac{1}{7} = 0.\dot{1}4285\dot{7}$$

$$= 0.\underbrace{142857}\,\underbrace{142857}\,\underbrace{142857}\cdots\cdots$$

$\times 2$ ← イコール

$$\frac{2}{7} = 0.\overbrace{285714}\,\overbrace{285714}\,\overbrace{285714}\cdots\cdots$$

$142857 \times 1 = 142857$

$142857 \times 3 = 428571$

$142857 \times 2 = 285714$

$142857 \times 6 = 857142$

$142857 \times 4 = 571428$

$142857 \times 5 = 714285$

$1 = \frac{1}{7} \times 7$, $142857 \times 7 = 999999$

$\frac{1}{7} = 0.\underbrace{142857}\ \underbrace{142857}\ \underbrace{142857}\ \cdots\cdots$

$\times 7 = 11$

$1 \stackrel{?}{=} 0.\overline{999999}\ \overline{999999}\ \overline{999999}\ \cdots\cdots$

$\frac{1}{17} = 0.\dot{0}5882352941176\dot{4}7$

0588235294117647 × 1 = 0588235294117647
0588235294117647 × 2 = 1176470588235294
0588235294117647 × 3 = 1764705882352941
0588235294117647 × 4 = 2352941176470588
0588235294117647 × 5 = 2941176470588235
0588235294117647 × 6 = 3529411764705882
0588235294117647 × 7 = 4117647058823529
0588235294117647 × 8 = 4705882352941176
0588235294117647 × 9 = 5294117647058823
0588235294117647 × 10 = 5882352941176470
0588235294117647 × 11 = 6470588235294117
0588235294117647 × 12 = 7058823529411764
0588235294117647 × 13 = 7647058823529411
0588235294117647 × 14 = 8235294117647058
0588235294117647 × 15 = 8823529411764705
0588235294117647 × 16 = 9411764705882352

0588235294117647 × 17 = 9999999999999999

$$\frac{1}{13} = 0.\dot{0}76923\dot{}$$

```
  0.076923
13)1
   10
    0
   100
    91
     90
     78
     120
     117
       30
       26
        40
        39
         1
```

$076923 \times 1 = 076923$

$076923 \times 10 = 769230$

$076923 \times 9 = 692307$

$076923 \times 12 = 923076$

$076923 \times 3 = 230769$

$076923 \times 4 = 307692$

$$\frac{2}{13} = 0.\dot{1}5384\dot{6}$$

```
  0.153846
13)2
  20
  13
   70
   65
    50
    39
     110
     104
      60
      52
       80
       78
        2
```

$$076923 \times 2 = 153846$$

$$076923 \times 7 = 538461$$

$$076923 \times 5 = 384615$$

$$076923 \times 11 = 846153$$

$$076923 \times 6 = 461538$$

$$076923 \times 8 = 615384$$

$076923 \times 13 = 999999$

$\frac{1}{7} = 0.\dot{1}4285\dot{7}$

$6 = 7 - 1$

$\frac{1}{17} = 0.\dot{0}5882352941176\dot{4}7$

$16 = 17 - 1$

$\frac{1}{13} = 0.\dot{0}76923\dot{}$

$6 = (13 - 1) \div 2$

定理　Pを2,5以外の素数としたとき，

$\frac{1}{P}$を循環小数で表したときの循環部分の長さは$P-1$の約数である。

未解決問題　循環部分の長さが$P-1$となる素数Pは無限個あるか？